Coded-Protocols for Implementing Cooperative Diversity

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Problem Statement

- Consider the following ad hoc network:

- Questions:
  - How can the message be efficiently routed to the destination?
  - What is the tradeoff between latency and energy efficiency?
  - How to implement error control, routing, and access control?
  - Hybrid ARQ = Both FEC and ARQ are permitted.
  - Joint (cross-layer) solution is emphasized.
Conventional Approach: Multihop

- Multihop picks from among several possible routes:

  ![Multihop Network Diagram]

- **Drawbacks**
  - Routing tables need to be created and maintained.
    - Not robust to changes in topology, interference, or channel.
  - Routing ultimately relies on cascade of point-point links.
    - Spatial (MIMO) diversity not exploited.
    - Wireless is broadcast-oriented, not link-oriented!
  - The network could instead be interpreted as a large array.

Conventional Antenna Arrays

- With a **conventional** array, elements are closely spaced ($\lambda/2$) and connected through high bandwidth cabling.
  - **Microdiversity.**
Distributed Antenna Array

With a **distributed** array, the antennas are widely separated (e.g. different base stations) and connected through a moderate bandwidth backbone.

- **Macrodiversity.**

![Distributed Antenna Array Diagram]

Virtual Antenna Array

With a **virtual** array, the antenna elements are widely spaced (attached to different receivers) but are **not** connected by a backbone.

- Virtual connection achieved by MAC-layer design.
- **Decentralized macrodiversity.**
- Related to the relay channel and cooperative diversity.

![Virtual Antenna Array Diagram]
Related Work

- Several options for exploiting the broadcast nature of radio have been proposed.

  - The relay channel (Cover/El Gamal 1979)
  - Cooperative diversity (Sendonaris/Erkip/Aazhang & Laneman/Wornell 1998)
  - Cooperative coding (Hunter & Nosratinia)
  - Parallel relay channel (Gatspar/Kramer/Gupta 2002)
  - Multihop diversity (Boyer/Falconer/Yanikomeroglu & Gupta/Kumar 2001)

Orthogonal Single-Relay Channel

- Assume block fading.
  - For one block of data, each channel is AWGN with instantaneous SNR $\gamma$
  - The SNRs change from block-to-block.
  - The average SNR is $\Gamma$.

  - A single channel is in an outage if:
    $$C(\gamma) = \frac{1}{2} \log_2(1 + \gamma) < r$$
    $r$ is code rate

  - The overall relay channel is in an outage if either:
    1. Both source-relay and source-destination link in outage:
       $$\left( C(\gamma_{s,r}) < \frac{r}{\alpha} \right) \cap \left( C(\gamma_{r,d}) < \frac{r}{\alpha} \right)$$
       $\alpha$ is fraction of time the source transmits
    2. Source-relay link not in outage but parallel link from relay and source to destination is in an outage:
       $$\alpha C(\gamma_{r,d}) + (1 - \alpha) C(\gamma_{s,d}) < r$$
Outages

- The outage event region is the range of instantaneous SNRs such that:

\[ R_o = \left[ \left( C(Y_{s,r}) < \frac{P_r}{\sigma^2} \right) \cap \left( C(Y_{r,d}) < \frac{P_r}{\sigma^2} \right) \right] \cup \left[ \left( C(Y_{s,r}) > \frac{P_r}{\sigma^2} \right) \cap \left( \alpha C(Y_{r,d}) + (1-\alpha) C(Y_{s,d}) < \frac{P_r}{\sigma^2} \right) \right] \]

- The outage event probability (OEP) is:

\[
P_o = \iiint_{k_d} f(Y_{s,r}, Y_{r,d}, dY_{s,r}, dY_{r,d}) dY_{s,r} dY_{r,d}
\]

\[
= \frac{1}{\Gamma_{s,r} \Gamma_{r,d}} \iiint_{k_d} \exp \left( \frac{Y_{s,r}}{\Gamma_{s,r}} + \frac{Y_{r,d}}{\Gamma_{r,d}} + \frac{Y_{s,d}}{\Gamma_{s,d}} \right) dY_{s,r} dY_{s,d} dY_{r,d}
\]

- Under the assumption of independent quasi-static Rayleigh fading channels.

Numerical Results

- Consider the following example:
  - The received power \( P_r \) at distance \( d_m \) is related to transmitted power \( P_t \) by

\[
P_r = \left( \frac{c}{4\pi d_m f_c} \right)^n \left( \frac{d_m}{d_s} \right)^n P_t = 10^{-4} d_m^n P_t
\]

  - Where \( f_c = 2.4 \text{ GHz} \), \( d_o = 1 \text{ m} \), and path loss coefficient \( n = 3 \).
  - Define the “transmit” SNR as \( P_t/(WN_o) \)

- We can visualize performance in two dimensions by plotting contours of source/relay transmit SNRs required to achieve desired OEP.

- Assume source & destination separated by 10 m
  - Relay lies on line connecting source & destination.
The Outage Event Probability (OEP)

![Graph showing the outage event probability (OEP) with different configurations of source, relay, and destination distances, and their corresponding average Transmit SNR values.]

Distributed Turbo Coding

- Source & relay each have a recursive encoder.
- If relay interleaves between decoding and re-encoding, then a turbo code has been created.

![Diagram illustrating the distributed turbo coding system with components labeled: Source, Relay, Destination, Interleaver, Turbo Decoder, Source-Relay Channel (RSC #1), and Relay-Destination Channel (RSC #2).]

Performance of Distributed Turbo Coding

frame size = 512 data bits
BPSK modulation

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Scaling to Large Networks

- The results can be extended to the multiple relay channel.
  - Could use a multiple turbo code.
  - Parallel relays or inter-relay communications.
  - However, scheduling with multiple relays is difficult.
- Another option is to use use generalized hybrid-ARQ.
  - Hybrid-ARQ
    - Encode data into a low-rate $R_m$ code
      - Implemented using rate-compatible puncturing.
    - Break the codeword into $M$ distinct blocks
      - Each block has rate $R = R_m/M$
    - Source begins by sending the first block.
      - If destination does not signal with an ACK, the next block is sent.
        - After $m$th transmission, effective rate is $R_m = R/m$
  - Generalized hybrid-ARQ
    - The retransmission could be from any relay that decoded the message.
    - In large network, relays form a subset of the network called a cluster.

Info Theory of Hybrid-ARQ

- Throughput of hybrid-ARQ has been studied by Caire and Tuninetti (IT 2001).
  - Let $\gamma_m$ denote the received SNR during the $m$th transmission
  - The instantaneous capacity is:
    \[
    C(\gamma_m) = \frac{1}{2} \log_2 (1 + \gamma_m)
    \]
  - The cumulative capacity is:
    \[
    C_m = \begin{cases}
      \sum_m C(\gamma_m) & \text{diversity combining} \\
      \sum_m \gamma_m & \text{code combining}
    \end{cases}
    \]
  - An outage occurs if
    \[
    C_m < R
    \]
HARBINGER

- Source broadcasts first packet, \( m=1 \).
- Relays that can decode are added to the decoding set \( D \).
  - The source is also in \( D \)
- The next packet is sent by a node in \( D \).
  - The choice of which node depends on the protocol.
  - Geographic-Relaying: Pick the node in \( D \) closest to destination.
- The process continues until the destination can decode.
- We term this protocol “HARBINGER”
  - Hybrid ARq-Based INtercluster GEographic Relaying.
- Energy-latency tradeoff can be analyzed by generalizing Caire and Tuninetti’s analysis.

HARBINGER: Initialization

Solid circles are in the decoding set \( D \).
Amount of fill is proportional to the accumulated entropy.
Keep transmitting until Destination is in \( D \).
HARBINGER: First Hop

HARBINGER: Selecting the Relay for the Second Hop
HARBINGER: Second Hop

HARBINGER: Third Hop
HARBINGER: Fourth Hop

HARBINGER: Results

Topology:
Relays on straight line
S-D separated by 10 m

Coding parameters:
Per-block rate R=1
No limit on M
Code Combining

Channel parameters:
\( n = 3 \) path loss exponent
2.4 GHz
\( d_0 = 1 \) m reference dist

Monte Carlo Integration

B. Zhao and M. C. Valenti. “A block-fading perspective on energy efficient random access relay networks”, to appear in JSAC special issue on Wireless Ad Hoc
Discussion

- Advantages.
  - Better energy-latency tradeoff than multihop.
    - Nodes can transmit with significantly lower energy.
    - System exploits momentarily good links to reduce delay.
  - No need to maintain routing tables (reactive).
- Disadvantages.
  - More receivers must listen to each broadcast.
    - Reception consumes energy.
  - Nodes within a cluster must remain quiet.
  - Longer contention period in the MAC protocol.
  - Results are intractable, must resort to simulation.
  - Requires position estimates.

These tradeoffs can be balanced by properly selecting the number of relays in a cluster.

Simplifying Assumptions

- Closed-form analysis is not tractable.
  - Statistically variable channels.
  - Nodes have memory for entire source-destination transaction.
  - Possible changes in topology.
- Analysis is possible under simplifying assumptions:
  - Channels are static (AWGN) for duration of transaction.
  - Nodes flush memory once a new relay is selected.
    - Still maintain memory of ARQ packets from current transmitter.
  - Topology is 2-D Poisson.
GeRaF

Geographic Random Forwarding (GeRaF)
- Node activity follows a sleep schedule.
  - Common strategy for sensor networks.
- Source broadcasts over an AWGN channel.
  - If one node is within range it becomes the designated relay.
  - If multiple nodes, the one closest to destination becomes relay.
  - Otherwise, source tries again later to see if a relay awoke.
  - No ARQ or diversity combining effect.
- This is precisely HARBINGER with the simplifying assumptions and M=1 (no ARQ)

HARBINGER: Simplified Analysis

Topology:
2-D Poisson
S-D separated by 10 m

Coding parameters:
Per-block rate R=1
Code Combining
Normalized power
(Initial TX range is 1 m)

Channel parameters:
n = 3 path loss exponent
2.4 GHz
d_0 = 1 m reference dist

Only requires calculating areas of the geographically advantaged regions

B. Zhao and M. C. Valenti. “Position-based relaying with hybrid-ARQ for efficient ad hoc networking,” submitted to EURASIP issue on Ad Hoc Networks.
Coding for HARBINGER

- To implement HARBINGER, need a code suitable for:
  - Hybrid ARQ: Rate compatible puncturing.
  - Block fading.

- One option is a carefully designed LDPC code.
  - Oftentimes LDPC codes are designed using girth-conditioning techniques.
    - Avoid 4-cycles.
    - Try to maximize the girth of the underlying Tanner graph.
  - For puncturing and for fading, **stopping sets** are more important.
    - **Stopping set**: A set of code symbols which, if all are erased, will not allow the decoder to converge.
    - Want to avoid small stopping sets.
    - Want to avoid all bits in a stopping set in same frame.

Distributed Space-Time Coding

- A source and relay in the decoding set could transmit concurrently using a space time code.

- There are several practical problems:
  - Lack of timing synchronism.
  - Frequency offset.

\[\text{Source} \quad \xrightarrow{\text{Relay}} \quad \text{Destination}\]

---, “Macroscopic space-time coding: Motivation, performance criteria, and a class of orthogonal designs”. CISS 2003
Conclusions

- Wireless is a broadcast-oriented medium
  - Link-oriented protocols do not exploit this.
- Cooperative diversity (orthogonal relaying) can give a better tradeoff between energy and latency.
  - The number of participating relays should be carefully chosen.
- A cross-layer approach can yield significant gains:
  - Error control using hybrid-ARQ
  - CSMA-style medium access control
  - Position-based relaying