POSITION DEPENDANT POWER ALLOCATION STRATEGIES IN COOPERATIVE RELAY NETWORKS

Somak Datta Gupta and Daryl Reynolds
West Virginia University
Morgantown, WV
Somak.DattaGupta@mail.wvu.edu, Daryl.Reynolds@mail.wvu.edu

ABSTRACT

In fading wireless channels, relays are often used with the aim of achieving diversity and thus overall performance gain. We use amplify and forward (AF) and decode and forward (DF) at the relay and optimize the power allocated to the relay, under total transmit power constraint, to maximize the instantaneous SNR and thus achieve improved throughput and BER performance. We compare the performance of the AF and DF protocols based on their positional BER and throughput and notice that our power optimized schemes outperform existing ones at certain areas. Finally, we also identify the region where relaying would provide performance gains for both the protocols.

INTRODUCTION

In cooperative relay network various forwarding techniques like amplify and forward and decode and forward are used at the relay for better performance. In a power constrained environment, the performance can be bettered by using an optimal power allocation strategy. The relative position of the relay with respect to the source and destination also has an immense effect on the efficacy of the relay. We position the relay at various positions in a planar grid, with the position of source and destination being fixed. Analytical expressions of the instantaneous SNR at the destination are derived for both AF and DF. These expressions are numerically optimized to obtain an optimum power allocation strategy for each position of the relay in both the AF and DF schemes. Previous works like [1], [2], [3], [4], [5] involving power allocation schemes have mainly concentrated on optimizing information theoretic quantities like capacity and outage probability. In [6] a power allocation scheme is proposed for the more complex adaptive decode and forward protocol where the destination has to know when the relay is forwarding, the source must have knowledge on the decoding status of the relay and there may also be requests for retransmission with the source repeating the message. Another power allocation scheme is proposed in [7], where the approach is from an information theoretic point of view with the aim of maximizing SNR and hence channel capacity, for a relay system in fading environment with instantaneous and mixed channel state information (CSI) at the transmitter. In [8] the author used a group of 48 × 48 relays in a square grid for evaluating cooperative relaying under power constraint, but in [9] it has been shown that in a relay network, data should be transmitted over a single relay path which is in the best position in the network. We investigate the effect that the positioning of the relay has on a relaying system and use our three terminal model, described in detail in the system model, to optimize the power allocation while using AF and DF protocols and coherent demodulation at the relay and destination. As practically the performance of any system ultimately depends on the instantaneous SNR and BER and their relationship, we derive expressions for instantaneous SNR using our model and optimize the power allocation based on that, with the final aim of achieving improved BER. We compare the performance of AF and DF using our model and also identify the regions where relaying is necessary and where it is not. The contributions of this paper are:

1. Formulating a position dependant power optimization for both AF and DF and showing that it outperforms arbitrary power allocations as well as some information theoretic approaches.

2. Identifying the cardioid shaped area about the source and destination where relaying using our power optimized AF offers performance gain. For DF protocol using our power optimization, the elliptical area where relaying is useful is also identified. In both these regions there is an improvement of end to end BER performance over direct transmission and this is bolstered by optimally allocating power depending on the relay position and also taking into account the path loss involved.

3. Quantifying the performance difference of
power optimized AF and DF relaying depending on the position of the relay.

SYSTEM MODEL

To simplify and facilitate the problem of power allocation, a single relay and half-duplex system model is used. Our system is a simple three terminal model as shown in Fig. 1 with one source, one destination and one relay denoted by s, d and r respectively. The system is half-duplex and the transmission from the source and that from the relay is assumed to be time-orthogonal. The modulation used is QPSK. Our system is uncoded and when using DF perfect CRC is assumed at the relay, so that the relay forwards only when it is able to decode correctly. Full CSI at relay and the receiver is assumed and coherent demodulation is performed. The channel is modeled as frequency non-selective with Rayleigh fading. As shown in Fig. 1 the channel gain between the source and receiver is denoted by $h_{s,d}$, the channel gain between the source and the relay by $h_{s,r}$ and that between relay and receiver by $h_{r,d}$. The channel gains are complex Gaussian with zero mean and unit variance. The distance between the source and the destination, denoted by $d_{s,d}$, is normalized to one and we also assume a fixed noise power. The distances between the source and relay and that between the relay and destination are represented as $d_{s,r}$ and $d_{r,d}$ respectively. The transmission is two phased:

**Phase 1** - The source transmits with a certain power, $p_1$ (indicated by the double lines in Fig. 1) and due to the broadcast nature of wireless communication, the relay as well as the destination receives a copy of it.

**Phase 2** - The relay forwards the received signal using either AF or DF protocol using power $p_2$ (indicated by the single line in Fig. 1). The two copies of the signals are then combined using maximal ratio combining and decoding.

To facilitate the determination of optimal power division between the two phases, we constrained the total power to one. This is summarized in (1) as,

$$P_{Total} = p_1 + p_2,$$

with $P_{Total}$ set to 1 so that we can effectively express $p_2$ as

$$p_2 = 1 - p_1.$$  

To investigate the effect of the relay position on the overall performance, the pathloss is also taken into consideration. We use a pathloss exponent of 3.6 and fix the noise power, $\sigma^2$ to $5 \times 10^{-15}$ Watts as used in [10]. A constant $k$ is introduced to flexibly scale the predetermined received SNR. We would consider a predetermined received SNR of 10dB throughout the paper. For the transmitted symbol $b$ the received signal $r$ can be expressed as

$$r = \frac{\sqrt{kh_b}}{d^{3.6}} + n.$$  

In (3), $h$ represents the channel gain or the fading coefficient and $n \sim \mathcal{N}(0, \sigma^2)$, the AWGN noise with the aforementioned power. The distance between the source and the receiver is quantified by $d$, normalized to one in our case and $p$ denotes the transmit power for that particular transmission phase. In this paper we have assumed that the relay uses either non-regenerative forwarding like AF or regenerative forwarding like DF. We evaluate the performance over a 3 unit by 3 unit grid, with the source at (-0.5, 0) and the destination placed at (0.5, 0). The relay can be placed anywhere within the grid.

**AMPLIFY AND FORWARD**

The simplicity of this kind of forwarding and its ease of implementation has lead to a lot of analysis of its various performance metrics. In this technique the relay almost act as a repeater. The signal received at the relay in Phase 1, from the source, is scaled and then retransmitted towards the receiver in Phase 2. We have assumed a total power constraint, so according to (2) in Phase 2 the relay uses the portion of power left over from the first phase. We would proceed to formulate the instantaneous combined received SNR for a predetermined received SNR which is used to calculate the probability of bit error. In Phase 1 the relay and the destination receive signals as given by (4) and (5) respectively:

$$r_{s,r} = \frac{\sqrt{p_1kh_{s,r}b}}{d^{3.6}_{s,r}} + n_{s,r}$$

$$r_{s,d} = \frac{\sqrt{p_1kh_{s,d}b}}{d^{3.6}_{s,d}} + n_{s,d}.$$
Before transmitting in Phase 2 the relay scales $r_{s,r}$ by multiplying it by $\beta$, which is given by,

$$\beta = \frac{1}{|r_{s,r}|}.$$  

At the end of Phase 2 the destination receives a noisy copy of the signal from the relay given by

$$r_{r,d} = \sqrt{P_2 kh_{r,d}^2 r_{s,r}} + n_{r,d}$$

$$= \sqrt{P_1 P_2 k^2 \beta h_{s,r} h_{r,d} b} \frac{d_{s,r}}{d_{s,d}^4 d_{r,d}^4} + \sqrt{P_2 k \beta h_{r,d} n_{s,r}} + n_{r,d}. \tag{8}$$

We now form the decision statistic as shown in (9) and from that derive the instantaneous SNR. To achieve this the signals $r_{s,d}$ and $r_{r,d}$ are individually multiplied by the complex conjugates of the respective channel gains, given by $h_{s,d}^*$ and $(h_{s,r} h_{r,d})^*$ and then combined to give

$$d_{AF} = h_{s,d}^* r_{s,d} + h_{s,r} h_{r,d}^* r_{r,d}$$

$$= h_{s,d}^* \left( \sqrt{P_1} k h_{s,d} b \frac{d_{s,d}^4}{d_{s,r}^4} + n_{s,d} \right) +$$

$$(h_{s,r} h_{r,d})^* \left( \sqrt{P_1 P_2 k^2 \beta h_{s,r} h_{r,d} b} \frac{d_{s,r}^4}{d_{s,d}^4 d_{r,d}^4} + \sqrt{P_2 k \beta h_{r,d} n_{s,r}} + n_{r,d} \right). \tag{9}$$

We can simplify (9) as

$$d_{AF} = \sqrt{P_1} |h_{s,d}|^2 b \frac{d_{s,d}^4}{d_{s,r}^4} + \sqrt{P_1 P_2 k^2 \beta |h_{s,r} h_{r,d}|^2 b} \frac{d_{s,r}^4}{d_{s,d}^4 d_{r,d}^4} +$$

$$h_{s,d}^* n_{s,d} + \sqrt{P_2 k \beta h_{r,d} n_{s,r}} + (h_{s,r} h_{r,d})^* n_{r,d}. \tag{10}$$

From (10), we express the instantaneous SNR as

$$\gamma_{AF} = \frac{\left( \sqrt{P_1} |h_{s,d}|^2 b \frac{d_{s,d}^4}{d_{s,r}^4} + \sqrt{P_1 P_2 k^2 \beta |h_{s,r} h_{r,d}|^2 b} \frac{d_{s,r}^4}{d_{s,d}^4 d_{r,d}^4} \right)^2}{|h_{s,d}|^2 + (\frac{k^2 \beta |h_{s,r} h_{r,d}|^2}{d_{r,d}^4} + |h_{s,r} h_{r,d}|^2) N_0}. \tag{11}$$

Using (2), (11) can be written as

$$\gamma_{AF} = \frac{\left( \sqrt{P_1} |h_{s,d}|^2 b \frac{d_{s,d}^4}{d_{s,r}^4} + \sqrt{P_1 P_2 k^2 \beta |h_{s,r} h_{r,d}|^2 b} \frac{d_{s,r}^4}{d_{s,d}^4 d_{r,d}^4} \right)^2}{|h_{s,d}|^2 + \left( \frac{1 - P_1 k^2 \beta |h_{s,r} h_{r,d}|^2}{(1 - d_{s,d}^4)} \right) + |h_{s,r} h_{r,d}|^2) N_0}. \tag{12}$$

Thus the average probability of bit error is

$$P_e = E[Q(\sqrt{\gamma_{AF}})]. \tag{13}$$

It is evident that if we are able to optimize the instantaneous phase powers to maximize $\gamma_{AF}$ and use the average of the optimized powers, we would be able to extract almost optimal BER performance from our system. We numerically optimize (12) for $P_1$, the power in Phase 1, using Quasi-Newton method, [11], under the total power constraint given by (11).

Fig. 2 shows the BER performance of the system which also shows that the best performance is achieved if the relay is placed about halfway between the source and destination, as has been shown.
tacitly from information theoretic angle in [12] and [7]. Fig. 3 points to fact that significant improvement over direct transmission is achieved over a wide area of the grid we considered. The inner lighter colored, cardioid shaped area represents the area where relaying is useful in AF.

![Figure 4: Phase 1 power $p_1$ for AF.](image)

The received SNR is 10dB. Source at x,y = (-0.5,0) and destination at x,y = (0.5,0). The z axis represents the value of $p_1$.

The portion of total power used for transmission in Phase 1 given by $p_1$ is plotted in Fig. 4 which shows how $p_1$ changes with the position of the relay. We see that in the region where the system BER performance is better we can use comparatively lesser amount of power in Phase 1.

**DECODE AND FORWARD**

Regenerative protocols such as DF are more complex to implement than non-regenerative protocols as AF but are of interest as they offer performance gains over the other. As in the case of AF the relay receives the broadcast from the transmitter in Phase 1, but instead simply scaling and forwarding the signal, the relay tries to decode the signal and then forward it. We assume perfect CRC in the source to relay link so that the relay can perform a CRC check on the received signal and forward only if it decodes correctly. So in this case the instantaneous SNR of the source to relay link assumes importance. In Phase 1 the relay and the destination receives signals given by (4) and (5). We model the probability that relay decodes correctly as a Bernoulli random variable

$$\eta \sim \text{Bernoulli}(\rho),$$  \hspace{1cm} (14)

whose parameter $\rho$ is the probability of receiving correct symbols from the source at the relay. In Phase 2 the relay transmits with the power $p_2$, the portion of power left over from Phase 1, given by (2). The signal received by the destination in Phase 2 can be expressed as

$$r_{r,d} = \left( \frac{\sqrt{2}\kappa h_{r,d}\eta b}{d_{r,d}^{3.6}} + n_{r,d} \right),$$  \hspace{1cm} (15)

where $\eta$ is the bernoulli random variable discussed in (14). From (15) it can be seen that whenever the relay determines it has not decoded correctly, the first term is zero and the signal received at the destination is pure noise. But the destination has no way to figure that and hence it always listens to the relay even when the relay is silent. An immediate effect is that the performance of power optimized relaying does not converge to that of direct transmission when the relay is not actually forwarding. The two received signals from the source in Phase 1 and the relay in Phase 2 are then multiplied by the complex conjugate of their individual channel gains and then combined to form the decision statistic as shown in (16).

$$d_{DF} = h_{s,d}^* r_{s,d} + h_{r,d}^* r_{r,d}$$

$$= h_{s,d}^* \left( \frac{\sqrt{\kappa} h_{s,d} b}{d_{s,d}^{3.6}} + n_{s,d} \right) + h_{r,d}^* \left( \frac{\sqrt{\kappa} h_{r,d} \eta b}{d_{r,d}^{3.6}} + n_{r,d} \right)$$

$$= \sqrt{p_1} k |h_{s,d}|^2 b \frac{1}{d_{s,d}^{3.6}} + h_{s,d}^* n_{s,d} + \frac{\sqrt{p_2} \kappa |h_{r,d}|^2 b}{d_{r,d}^{3.6}} + h_{r,d}^* n_{r,d}. \hspace{1cm} (16)$$

From (16) we can write the instantaneous SNR as

$$\gamma_{DF} = \frac{\left( \sqrt{p_1} k |h_{s,d}|^2 + \sqrt{p_2} \kappa |h_{r,d}|^2 \right)^2}{(|h_{s,d}|^2 + |h_{r,d}|^2) N_0}. \hspace{1cm} (17)$$

Using (2) and noting that the source to destination distance is one, we can write (17) as

$$\gamma_{DF} = \frac{\left( \sqrt{p_1} k |h_{s,d}|^2 + \sqrt{1-p_1} \kappa |h_{r,d}|^2 \right)^2}{(|h_{s,d}|^2 + |h_{r,d}|^2) N_0}. \hspace{1cm} (18)$$

Similar to the AF case the probability of bit error is expressed as

$$P_e = E[\text{Q}(\sqrt{\gamma_{DF}})]. \hspace{1cm} (19)$$

Arguing as before, we proceed to numerically optimize (18) for the Phase 1 power, $p_1$ using Quasi-Newton method under total power constraint as before.

Fig. 5 shows again that the best performance is achieved the positioning the relay about midway
BER performance of DF. The received SNR is 10dB. Source at x,y = (-0.5,0) and destination at x,y = (0.5,0).

Figure 5: BER performance of DF. The received SNR is 10dB. Source at x,y = (-0.5,0) and destination at x,y = (0.5,0).

The inner lighter area represents the region for relaying in DF. The received SNR is 10dB. Source at x,y = (-0.5,0) and destination at x,y = (0.5,0).

Figure 6: The inner lighter area represents the region for relaying in DF. The received SNR is 10dB. Source at x,y = (-0.5,0) and destination at x,y = (0.5,0).

that the allocated power for AF and DF is very similar.

SIMULATION RESULTS

Having established the basic framework we now proceed to analyze our results. We consider various arbitrary power allocations for both AF and DF and evaluate their performance assuming that the relay is allowed to be in any position along a straight line between the source and the destination which are unit distance apart. We have used notations like (10-90) to represent the case when 10 percent of the total power is used in Phase 1 and 90 percent in Phase 2 and so on. From Fig. 8 and Fig. 9 we see that using the power optimization technique we get the best overall performance, reiterating the importance of optimal power allocation.

In Fig. 10 we look at the performances of AF, DF and direct transmission over a grid. We notice that the DF protocol, wherein the relay transmits only when it decodes correctly, outperform the non-regenerative AF protocol, which forwards all the time. Not only is the performance of DF better than AF, it also has wider region where relaying is meaningful. This is also evident from Fig. 8 and Fig. 9. The throughput of DF is also better than that of the AF as shown in Fig. 11 which shows the throughputs of AF and DF using optimal power allocation when the relay is collinear with the source and destination. By throughput we will mean throughput per timeslot in this paper. Considering that we have a half duplex system, we get better throughput and BER for both AF and DF when the relay is in the midway region. From
Figure 8: BER performance of AF with arbitrary and optimal power allocation. The received SNR is 10dB. Source at $x,y = (0,0)$ and destination at $x,y = (1,0)$. AF(10-90) represents the case when 10 percent of the total power is used in Phase 1 and 90 percent in Phase 2 and so on.

Figure 9: BER performance of DF with arbitrary and optimal power allocation. The received SNR is 10dB. Source at $x,y = (0,0)$ and destination at $x,y = (1,0)$. DF(10-90) represents the case when 10 percent of the total power is used in Phase 1 and 90 percent in Phase 2 and so on.

Figure 10: BER performance of AF and DF with optimal power allocation and direct transmission. The received SNR is 10dB. Source at $x,y = (-0.5,0)$ and destination at $x,y = (0.5,0)$.

Figure 11: Comparison of throughput performance between [Qi et.al.] and our model for DF and AF with optimal power allocation. The received SNR is 10dB. Source at $x,y = (0,0)$ and destination at $x,y = (1,0)$.

Fig. 11 it is also seen that by using our optimized power allocation there are throughput gains over the method used in [7]. In [7] the authors have proposed a power allocation scheme from an information theoretic angle with the aim of maximizing the channel capacity and SNR in a relaying system using regenerative DF protocol. There the transmitter knows the instantaneous channel state information, which allows it to switch the relay off depending on the SNR of particular links.

From Fig. 12 we can see that our DF protocol, with optimized power, matches or outperforms theirs at most places, even though the relay in our system is never fully switched off. When we use the power allocation in [7] we see that in areas where the power used is a very small portion of the total power, there is a loss in throughput and degeneration of BER in those regions. This can be clearly seen in Fig. 11 and Fig. 12 between $d_{s,r} = 0.15$ and $d_{s,r} = 0.5$. 
CONCLUSION

By using our power optimized AF and DF, we identify the cardioid and elliptical shaped regions for AF and DF where relaying would provide better performance than direct transmission. It is also noticed that this region is greater for DF than AF i.e. for the same received SNR, DF has a wider area around the source and destination where relaying provides performance benefits than AF. Also in this region DF outperforms AF. Some of the non optimal schemes work differently in AF and DF at certain areas, but the optimal power allocation is very similar for both the protocols, pointing to the fact that there are certain powers which perform best in both AF and DF for a particular area. This, with further studies, might lead to a better understanding about the effect and importance of power allocation in cooperative relay networks. We conclude that our position dependant power optimization not only improves the system performance over arbitrary power allocations but also outperforms some information theoretic approaches. The areas of interest where relaying would be useful, while using our power optimization scheme, are also identified for both AF and DF.

REFERENCES


