ADVANCED SIGNAL PROCESSING TECHNIQUES FOR INTERFERENCE SUPPRESSION IN CDMA COMMUNICATIONS

A Dissertation
by
DARYL S. REYNOLDS

Submitted to the Office of Graduate Studies of Texas A&M University in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2002

Major Subject: Electrical Engineering
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August 2002

Major Subject: Electrical Engineering
ABSTRACT

Advanced Signal Processing Techniques for Interference Suppression in CDMA Communications. (August 2002)
Daryl S. Reynolds, B.S., University of Colorado, Boulder; M.S., Texas A&M University
Chair of Advisory Committee: Dr. X. Wang

The ever increasing demand for performance and capacity in wireless systems has motivated the development of advanced signal processing techniques for signal reception. In this work, we develop, analyze, and simulate several signal processing techniques for the mitigation of interference in code-division multiple-access (CDMA) systems. In particular, we present our contributions to five areas: turbo receivers, space-time processing, low-complexity blind adaptive multiuser detection, transmitter optimization for blind and group-blind multiuser detection, and transmitter precoding. In each case, we will see that the algorithms presented offer significant advantages over existing techniques. We also present suggestions for future research.
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I would like to thank my committee chairman, Dr. Xiaodong Wang for his patience and guidance throughout this project. I would also like to thank all the members of my committee for their time and effort.
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CHAPTER I

INTRODUCTION

Code-division multiple-access (CDMA) is perhaps the most promising of the third and future generation cellular multiple-access formats [1]. Among its chief benefits are flexibility, high capacity potential, and an intrinsic resistance to narrowband interference. Despite these advantages, however, there remain significant challenges that must be addressed in the design and implementation of any practical CDMA system. Among the physical layer impediments that must be overcome are multiple-access interference (MAI) and dispersive channel impairments, including multipath and intersymbol interference. In this work, we will develop and analyze signal processing algorithms and tools that address MAI and dispersive channel interference in CDMA systems. In this chapter we will introduce five particular areas of study and summarize the results that appear in the rest of this work.

A. Turbo Processing

One of the most powerful signal processing techniques developed in recent years for interference suppression in direct-sequence spread-spectrum (DS-SS) and CDMA is iterative (turbo) processing. The inspiration behind turbo processing is the iterative procedure for the decoding of turbo coded systems, in which the received signal contains two-dimensional redundancy in the form of two recursive systematic convolutional (RSC) codes separated by an interleaver. Turbo decoding is accomplished via an iterative process in which (extrinsic) likelihood information is fed back and forth between the two RSC channel decoders. In the turbo processing techniques presented

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in this work, the systems are protected by a single non-recursive, non-systematic convolutional code; the second form of redundancy is induced by ISI or multipath interference or by the spread-spectrum modulation of the transmitted signal. The receiver consists of a single maximum a posteriori probability (MAP) channel decoder for extracting redundancy information from the code, and an MMSE module for extracting redundancy induced by the interference or modulation. Likelihood information (extrinsic information) is fed back and forth between the two receiver modules in an iterative fashion until convergence is achieved and performance improvement ceases.

In Chapter II we will introduce a new low-complexity turbo equalizer for single-user DS-SS systems in which diversity reception is available. We will see that this equalizer offers performance that is similar to that of existing turbo-equalization techniques, but with polynomial complexity rather than exponential complexity. In Chapter III we will develop a turbo multiuser detection algorithm for situations in which the receiver has knowledge of the spreading codes of only a subset of all the users in the system. This work extends [2] in which all spreading codes are assumed known. We will see that the proposed turbo group-blind receiver provides a significant performance gain over traditional turbo multiuser detection for situations in which some of the spreading codes are unknown.

B. Low-Complexity Blind Adaptive Multiuser Detection

In [3], Wang and Høst-Madsen developed group-blind multiuser detectors for use in CDMA uplink environments in which the base station receiver has knowledge of the spreading sequences of all the users within the cell, but no knowledge of the spreading sequences of users outside the cell. It is demonstrated that these detectors outperform the popular MMSE multiuser detector for situations in which some of the
spreading codes are unknown. Chapter IV we use the closed form expressions for these detectors, along with a blind Kalman channel estimator and a new, high-performance, low complexity subspace tracking algorithm to develop a blind adaptive group-blind multiuser detector that is able to detect and adapt to changes in the number of users and their composite signature waveforms in an asynchronous multipath environment. We present steady state performance as well as the ability of the receiver to track changes in the signal subspace. We will see that the performance of the blind adaptive group-blind detector is significantly superior to its blind counterpart when more than one signature sequence is known to the receiver.

C. Space-Time Multiuser Detection

Time diversity, in the form of channel coding, and frequency diversity, in the form of spread-spectrum modulation, have long been used to improve the performance and capacity of cellular systems. More recently, researchers have become interested in space-time processing, in which multiple transmit/receive antennas are used in conjunction with coding to exploit spatial diversity. Of particular interest are transmit diversity schemes in which multiple antennas are utilized at the transmitter. These schemes are appealing since they can be used to improve downlink performance without adding multiple antennas to a small mobile unit and because the additional costs are incurred primarily at the transmitter rather than the receiver. Although a significant amount of work has been completed in this area [4, 5, 6, 7, 8, 9], most of this work has one or more of the following limitations:

- a single antenna at the receiver or transmitter.
- an impractical channel model.
- high complexity.
• not applicable to CDMA with multiuser detection.

In Chapter V we develop and analyze a space-time multiuser detection algorithm that addresses these limitations. Specifically, we will compare, via analytical bit-error-probability calculations, user capacity, and complexity, two linear receiver structures for different antenna configurations. Motivated by its appearance in a number of 3G wideband CDMA standards, we use the Alamouti space-time block code for two-transmit-antenna configurations. We also develop blind adaptive implementations for the two transmit/two receive antenna case for synchronous CDMA in flat fading channels and for asynchronous CDMA in fading multipath channels. Finally, we present simulation results for the blind adaptive implementations.

D. Transmitter Optimization for Blind and Group-Blind Multiuser Detection

The linear subspace-based blind and group-blind multiuser detectors recently developed represent a robust and efficient adaptive multiuser detection technique for CDMA systems [10, 11, 3]. In Chapter VI we consider adaptive transmitter optimization strategies for CDMA systems in which these detectors are employed. We make use of more recent results on the analytical performance of these blind and group-blind receivers in the design and analysis of the transmitter optimization techniques [12, 13, 14, 15, 16, 17]. In particular, we will develop a maximum-eigenvector-based method of optimizing spreading codes for given channel conditions and a utility-based power control algorithm for CDMA systems with blind or group-blind multiuser detection. We also design a receiver incorporating joint optimization of spreading codes and transmitter power by combining these algorithms in an iterative Lloyd-Max type configuration. We will see that the utility-based power control algorithm allows us to efficiently set performance goals for users in heterogeneous traffic environments and
that spreading code optimization allows us to achieve these goals with lower transmit power. The algorithms presented in this chapter are distributed and maintain the blind (or group-blind) nature of the receiver.

E. Transmitter Precoding

The ever increasing demand for performance and capacity in cellular systems has prompted the development of increasingly sophisticated and complex signal processing techniques for signal reception. Many of these techniques impose a significant computational burden on the mobile receiver. However, the goal of maintaining low cost and complexity at the mobile unit is as important as ever. Hence, researchers have begun investigating signal processing techniques that move computational complexity from the mobile unit to the base station. In Chapter VII we propose and analyze a transmitter precoding strategy for the suppression of multiple-access interference and dispersive channel interference in a cellular CDMA system. More specifically, we develop a linear MMSE-based precoding strategy for fading multipath channels that allows for simple matched filtering at the mobile unit and is easy to make adaptive. We also present a performance analysis using tools developed for the analysis of conventional (receiver-based) linear blind multiuser detection in unknown channels. We compare the analytical and simulation results to traditional receiver-based blind multiuser detection. We will see that transmitter precoding offers a reasonable alternative for time division duplex mode CDMA when minimizing computational complexity at the mobile unit is a priority.
CHAPTER II

LOW COMPLEXITY TURBO-EQUALIZATION FOR DIVERSITY CHANNELS

A. Introduction

The increasing demand for high speed wireless products in urban and indoor environments has generated a great deal of interest in ways to combat the intersymbol interference (ISI) resulting from multipath transmission. In this chapter we will develop a low-complexity iterative receiver for severe, frequency selective ISI channels. The receiver has a two stage structure; the first stage is a soft-input soft-output (SISO) detector based upon the MMSE criterion, and the second stage is a symbol-by-symbol maximum a posteriori probability (MAP) channel decoder which generates optimum soft decisions for any code that can be represented by a trellis of finite duration. This structure was first proposed by Wang and Poor for multiuser detection in synchronous CDMA channels and in asynchronous multipath CDMA channels [2]. Other researchers have taken advantage of the Markov (trellis) structure of the discrete-time ISI channel and have developed iterative decoding methods using the SOVA or MAP as the channel equalizer [18, 19]. However, the computational complexity of these algorithms is exponential in the channel ISI length, $L$, and becomes prohibitive when the length is large (perhaps 10 or more). In contrast, the technique presented here has polynomial complexity in $L$, while its performance is comparable with the SOVA and MAP equalizers.

B. Turbo-Equalization Principle

As discussed briefly in Chapter I, the inspiration behind turbo-equalization is turbo decoding, in which the received signal contains two-dimensional redundancy in the
form of two recursive systematic convolutional (RSC) codes separated by an interleaver. Decoding is accomplished via an iterative process in which extrinsic information is fed back and forth between the two RSC channel decoders. Here we consider a system protected by a single non-recursive, non-systematic convolutional code; the second form of redundancy is the ISI created by the channel. The receiver structure is shown in Figure 1. The SISO detector delivers the \textit{a posteriori} log-likelihood ratio (LLR) of a transmitted “+1” and a transmitted “-1” for each code bit,

$$
\Lambda_1[b(i)] \triangleq \log \frac{\Pr[b(i) = +1 | r(t)]}{\Pr[b(i) = -1 | r(t)]} = \log \frac{p[r(t) | b(i) = +1]}{p[r(t) | b(i) = -1]} + \log \frac{\Pr[b(i) = +1]}{\Pr[b(i) = -1]}, \quad (2.1)
$$

where the second term in (2.1), denoted by \( \lambda_2^p[b(i)] \) represents the \textit{a priori} LLR of the code bit \( b(i) \) delivered by the channel decoder in the previous iteration (the superscript \( p \) indicates a quantity obtained from the previous iteration). The first term in (2.1), \( \lambda_1[b(i)] \), represents the \textit{extrinsic} information delivered by the SISO detector and is based upon the received signal \( r(t) \) and the prior information about the code bits other than the \( i \)-the bit, \( \lambda_2^p[b(j)], j \neq i \). The extrinsic information \( \lambda_1[b(i)] \), which is not influenced by \( \lambda_2^p[b(i)] \), the \textit{a priori} information provided by the channel decoder, is de-interleaved and fed into the channel decoder as the \textit{a priori} information in the next iteration. Using the prior information about all of the code bits and the trellis structure of the code, the SISO channel decoder provides the \textit{a posteriori} LLR of each code bit,

$$
\Lambda_2[b(n)] \triangleq \log \frac{\Pr[b(n) = +1 | \{\lambda_1^p[b(n)]\}_{n=0}^{M-1}; \text{decoding}]}{\Pr[b(n) = -1 | \{\lambda_1^p[b(n)]\}_{n=0}^{M-1}; \text{decoding}]} = \lambda_2[b(n)] + \lambda_1^p[b(n)]. \quad (2.2)
$$
where the second inequality is shown in [2]. It is clear from (2.2) that the output of the SISO channel decoder is the sum of the prior information provided by the SISO detector, $\lambda_1^p[b(n)]$, and the extrinsic information delivered by the channel decoder. This extrinsic information represents the information about $b(n)$ obtained from the prior information of the other code bits, $\lambda_1^p[b(m)], m \neq n$, and the trellis structure of the code. The channel decoder also computes the \textit{a posteriori} LLR of every information bit, which is used to make a decision on the decoded bits at the last iteration. After interleaving, the extrinsic information delivered by the channel decoder is passed to the SISO detector to be used as \textit{a priori} information in the next iteration. Since the ISI and the channel code are independent forms of redundancy, it is apparent that at the first iteration the extrinsic information $\lambda_1[b(i)]$ and $\lambda_2[b(i)]$ are statistically independent. Specifically, notice that for the first iteration $\lambda_2[b(i)]$ is a function of $\{\lambda_1[b(j)]\}_{j=0; j \neq i}^{M-1}$ which is a function of the received signal and of the prior information, which is not available for the first iteration. Hence, $\lambda_2[b(i)]$ has no statistical dependence on $\{\lambda_1[b(i)]\}$. After each further iteration, however, $\lambda_2[b(i)]$ becomes more and more correlated with $\{\lambda_1[b(i)]\}$ through the dependence of $\{\lambda_1[b(i)]\}$ on prior information. As a result, improvement through iteration diminishes.

C. System Description

1. Diversity ISI Signal Model

Following the development in [20], we consider a convolutionally coded system and assume the receiver is preceded by a matched filter followed by a noise whitening filter. The received signal is sampled at a multiple ($M$) of the symbol rate. The cascade of the pulse shape filter, the linear channel distortion, the whitening filter,
Fig. 1. Turbo-equalizer transmitter/receiver structure.

and $M$-times symbol rate sampling results in the discrete time model,

$$r(n) = \sum_{l=0}^{L-1} b(n-l)h(n;l) + \sigma v(n), \quad n = 0, 1, \ldots, N - 1, \quad (2.3)$$

where $N$ is the code bit block size, $r(n) = [r_0(n) \ r_1(n) \cdots r_{M-1}(n)]^T$ represents the output sequence vector at symbol interval $n$, $b(n) \in \{-1, +1\}$ are the code bits, $v(n) = [v_0(n) \ v_1(n) \cdots v_{M-1}(n)]^T$ is an $M$-vector of complex-valued i.i.d, zero-mean, Gaussian random variables with $E\{v_i(n)^2\} = 1$, and $h(n;l)$ are the channel tap weight coefficients. The channel ISI length is $L$. The tap coefficients are complex-valued processes, constant in the case of an additive Gaussian channel and Gaussian in the case of a Rayleigh fading channel. They are assumed perfectly known. Note that the vector ISI model (2.3) can also represent a multiple receive antenna system, where $M$ is the number of antennas. Furthermore, this model may represent a system which employs both multiple antenna and oversampling forms of diversity.
We define the following vectors and matrices:

\[
\mathbf{r}(n) \triangleq \begin{bmatrix}
  r(n + L - 1) \\
  \vdots \\
  r(n)
\end{bmatrix}_{ML \times 1}, \quad \mathbf{v}(n) \triangleq \begin{bmatrix}
  v(n + L - 1) \\
  \vdots \\
  v(n)
\end{bmatrix}_{ML \times 1},
\]

\[
\mathbf{b}(n) \triangleq [b(n + (L - 1)) \ b(n + (L - 2)) \cdots b(n) \ b(n - 1) \cdots b(n - (L - 1))]^T.
\]

\[
\mathbf{H}(n) \triangleq \begin{bmatrix}
  \mathbf{H}(n) & 0 & \cdots & 0 \\
  0 & \mathbf{H}(n) & \cdots & \vdots \\
  \vdots & \ddots & \ddots & 0 \\
  0 & \cdots & 0 & \mathbf{H}(n)
\end{bmatrix}_{ML \times (2L-1)}.
\]

Then (2.3) can be written in matrix form as

\[
\mathbf{r}(n) = \mathbf{H}(n)\mathbf{b}(n) + \sigma\mathbf{v}(n).
\]

The diversity factor \( M \) must be greater than or equal to 2 for \( \mathbf{H} \) to have full column rank. Consequently, this equalizer requires a diversity factor of at least 2.

2. SISO MMSE Detector

The MMSE detector is a modification of the detector developed in [2] for multiuser detection in multipath CDMA channels. Using the \textit{a priori} LLR of the code bits \( \{b(k)\}_{k=n-(L-1)}^{n+(L-1)} \), we form soft estimates of these bits as

\[
\tilde{b}(k) = \tanh \left( \frac{\lambda_2 \lambda[b(k)]}{2} \right), \quad n - (L - 1) \leq k \leq n + (L - 1).
\]

Let \( \tilde{\mathbf{b}}(n) \triangleq [\tilde{b}(n + (L - 1)) \ \tilde{b}(n + (L - 2)) \cdots \tilde{b}(n + 1) \ 0 \ \tilde{b}(n - 1) \cdots \tilde{b}(n - (L - 1))]^T \). For the code bit \( b(n) \), a soft interference cancellation is performed on the matched-filter
output $r(n)$ to obtain

$$\hat{r}(n) \triangleq r(n) - H(n)b(n) = H(n)[b(n) - \bar{b}(n)] + \sigma v(n)$$ \hspace{1cm} (2.10)$$

Next, an instantaneous linear MMSE filter $m(n)$ is applied to $\hat{r}(n)$ to obtain

$$z(n) \triangleq m(n)^H \hat{r}(n)$$ \hspace{1cm} (2.11)$$

where $m(n)$ is chosen to minimize the MSE between the code bit $b(n)$ and the filter output $z(n)$, i.e.

$$m(n) = \arg \min_{m \in \mathbb{C}^L} E \{ ||b(n) - m^H \hat{r}(n)||^2 \}$$ \hspace{1cm} (2.12)$$

$$= \arg \min_{m \in \mathbb{C}^L} m^H E\{\hat{r}(n)\hat{r}(n)^H\}m - 2m^H E\{b(n)\hat{r}(n)\}. \hspace{1cm} (2.13)$$

From (2.10) we have

$$E\{\hat{r}(n)\hat{r}(n)^H\} = H(n)\Lambda(n)H^H(n) + \sigma^2 I$$ \hspace{1cm} (2.14)$$

$$E\{b(n)\hat{r}(n)\} = H(n)e_L$$ \hspace{1cm} (2.15)$$

where $\Lambda(n) \triangleq \text{Cov}\{b(n) - \bar{b}(n)\} = \text{diag}(1 - \tilde{b}^2(n - (L - 1)), \ldots, 1 - \tilde{b}^2(n - 1), 1, 1 - \tilde{b}^2(n + 1), \ldots, 1 - \tilde{b}^2(n + (L - 1)))$ and $e_L$ is a vector of length $2L - 1$ whose elements are all zero except the $L$-th element which is 1. We may substitute (2.14) and (2.15) into (2.13) and write the MSE as

$$\text{MSE} = m^H[H(n)\Lambda(n)H^H(n) + \sigma^2 I]m - 2m^H H(n)e_L + 1.$$ \hspace{1cm} (2.16)$$

We take the gradient with respect to $m$ and set it to zero to obtain

$$[H(n)\Lambda(n)H^H(n) + \sigma^2 I]m - H(n)e_L = 0.$$ \hspace{1cm} (2.17)$$
Solving for \( m \), we obtain

\[
m(n) = [H(n)\Lambda(n)H^H(n) + \sigma^2 I]^{-1}H(n)e_L. \tag{2.18}
\]

Hence,

\[
z(n) = e_L^T H^H(n)[H(n)\Lambda(n)H^H(n) + \sigma^2 I]^{-1}[r(n) - H(n)\bar{b}(n)]. \tag{2.19}
\]

If we assume the output of the soft instantaneous MMSE filter represents the output of an equivalent AWGN channel having \( b(n) \) as its input symbol \([21]\), we may write

\[
z(n) = \mu(n)b(n) + \eta(n) \tag{2.20}
\]

where \( \mu(n) \) is the equivalent amplitude of the signal at the output and \( \eta(n) \) is a Gaussian random variable with mean zero and variance \( \nu^2(n) \). We may compute the parameters \( \mu(n) \) and \( \nu^2(n) \) as follows

\[
\mu(n) = E\{z(n)b(n)\} \tag{2.21}
\]

\[
= [H^H(n)[H\Lambda(n)H^H(n) + \sigma^2 I]^{-1}H(n)]_{L,L} \tag{2.22}
\]

and

\[
\nu^2(n) = \text{var}\{z(n)\} \tag{2.23}
\]

\[
= \mu(n) - \mu^2(n). \tag{2.24}
\]

The extrinsic information delivered by the soft instantaneous MMSE filter is given by

\[
\lambda_1[b(n)] \triangleq \log \frac{p(z(n)|b(n) = +1)}{p(z(n)|b(n) = -1)} = \frac{4\Re\{z(n)\}}{1 - \mu(n)}. \tag{2.25}
\]

The algorithm is summarized in Table I. The primary computational burden involved in finding the LLR at the \( n \)-th code bit interval is the matrix inversion
\( \Phi(n) \triangleq [H(n)A(n)H^H(n) + \sigma_2 I]^{-1} \). Since \( H(n)A(n)H^H(n) \) is the sum of \( 2L - 1 \) vector products, we may apply the matrix inversion lemma and compute \( \Phi(n) \) recursively [2]. The computational complexity of this Turbo-equalizer, per bit, is then \( O(M^2 L^2 + 2^\nu) \), where \( \nu \) is the code constraint length and \( M \) is the diversity factor discussed in Section 3.1.

**Table I**

**Turbo-equalization algorithm.**

<table>
<thead>
<tr>
<th>For ( n = 1, 2, \ldots, N - 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Given:</strong> ( r(l), b(l), l = 0, 1, \ldots, N - 1 ).</td>
</tr>
</tbody>
</table>

1. Form soft estimates of code bits:
   \[
   \tilde{b}(k) = \tanh \left[ \lambda_2^b[b(k)]/2 \right], \quad n - (L - 1) \leq k \leq n + (L - 1).
   \]

2. Perform soft interference cancellation on \( r(n) \):
   \[
   \hat{r}(n) \triangleq r(n) - H(n)\tilde{b}(n).
   \]

3. Apply soft MMSE filter to \( \hat{r}(n) \):
   \[
   z(n) \triangleq m(n)^H \hat{r}(n).
   \]

4. Calculate \( \mu(n) \):
   \[
   \mu(n) = \left[ H^H(n)[H A(n)H^H(n) + \sigma_2 I;^{-1} H(n)] \right]_{L,L}.
   \]

5. Compute the *a posteriori* LLR for code bit \( b(n) \):
   \[
   \lambda_1[b(n)] = 4 \text{Re}\{z(n)\} / [1 - \mu(n)].
   \]

end
3. SISO Channel Decoder

The input to the SISO channel decoder is the set of \textit{a priori} LLRs of all the code bits in the frame. It delivers, as output, updated LLRs of the code bits and, at the last iteration, the LLRs of the information bits. A forward-backward recursion of the type in [22] was developed in [2] and was used here. If a slight performance degradation is tolerable, one can reduce complexity by using the SOVA or Max-Log-Map algorithms to generate the required likelihood ratios. Bauch and Franz [23] have investigated the relative performance of these algorithms in the context of Turbo detection.

D. Simulation Results

In this section we investigate the performance of this Turbo-equalizer in Gaussian and Rayleigh channels via simulation. The form of diversity we chose to simulate is oversampling and the oversampling factor is $M = 2$. We assume BPSK modulation with a rectangular pulse shape employing a rate 1/2 convolutional code of constraint length $\nu = 3$ whose generators are $g_1 = [110]$ and $g_2 = [111]$. The channel ISI length, $L$, is 10 and the block size is 128 information bits. The interleaver is random and has a size equal to the code bit block size. The same interleaver is used throughout the simulations. In Figure 2 we see the performance for the first 4 iterations over a Gauss-type channel. The BER is plotted against $E_b/N_0$ where $E_b$ is the energy per information bit and $N_0$ is the noise power bilateral spectral density. For comparison, each of the figures contains a plot of the performance when the channel induces no ISI. We see that for 2 or more iterations the performance is very close to that obtained over a non-selective channel. Figures 3 and 4 show the performance when the channel coefficients are complex Gaussian processes. The normalized Doppler frequency, $BT$ Hertz/second, which is the product of the Doppler bandwidth and the
symbol duration, governs the rate of change of the channel. Figure 3 represents the performance for a relatively fast fading channel, with $BT = .1$ and Figure 4 shows the performance for a slowly fading channel, where $BT = .001$. Each path in the Rayleigh channel has equal mean power equal to $1/L$. We see that the Turbo-equalizer is unable to completely eliminate the ISI for a slowly varying channel, but performs well for a moderate or fast varying channels. This seemingly strange result was also reported in [18] and appears to be due to the fact that the same interleaver size is used for both fast and slow fading scenarios. Specifically, when very slow fading occurs, it can affect a number of symbols that approaches the length of the interleaver. In such cases, a longer interleaver is necessary to avoid performance degradation.

In summary, this receiver structure offers significant performance gain over the traditional, non-iterative receiver, whose performance is illustrated in each figure by the first iteration.

E. Conclusions

In this chapter we have presented an alternative method of Turbo-equalization in ISI diversity channels that offers performance similar to previous methods, but at lower computational complexity. Specifically, we have demonstrated that Turbo-equalization using the MMSE criterion virtually eliminates ISI in Gaussian and fast-varying Rayleigh channels and reduces dramatically, though does not eliminate, ISI in slow fading channels. The relatively low complexity of this algorithm allows its use in the severe ISI channels that often accompany high-data rate wireless applications.
Fig. 2. Performance of the turbo-equalizer over a Gaussian channel.
Fig. 3. Performance of the turbo-equalizer over a Rayleigh channel ($BT = .1$).
Fig. 4. Performance of the turbo-equalizer over a Rayleigh channel ($BT = .001$).
CHAPTER III

TURBO MULTIUSER DETECTION WITH UNKNOWN INTERFERERS

A. Introduction

Most of the early work on multiuser detection for code-division multiple-access (CDMA) focused on uncoded systems. Since most practical CDMA systems use error control coding, more recent work has addressed coded systems. Optimal joint decoding and symbol detection for coded asynchronous CDMA, for example, was investigated in [24]. However, the computational complexity resulting from the combined trellises of the convolutional code and the multiuser detector is $O(2^{K\nu})$ where $K$ is the number of users and $\nu$ is the constraint length of the code. Some suboptimal techniques that separate the functions of symbol detection and channel decoding are studied in [25].

More recently, iterative (Turbo) receivers for coded CDMA have received much attention. The inspiration behind these receivers is the decoding of turbo codes [26, 27], in which the transmitted signal contains two-dimensional redundancy in the form of two recursive, systematic convolutional codes separated by an interleaver. Decoding is accomplished via an iterative process in which extrinsic likelihood information is fed back and forth between two soft-input soft-output channel decoders. Turbo receivers for CDMA typically use only a single convolutional code. The second form of redundancy is induced by the channel (in the form of ISI, multipath, etc.) or by the structure of the transmitted signal. Examples of turbo equalization for single-user convolutionally coded transmission over intersymbol interference (ISI) channels include [18, 19, 28]. Turbo multiuser detection methods based on different interference cancellation schemes are proposed in [29, 30, 31, 32, 33]. In [34], Moher develops an optimal iterative multiuser detector for synchronous coded CDMA based on cross-
entropy minimization. Reed et al. [35] developed an iterative receiver that has some similarities to [18] but whose application is to turbo-coded CDMA. These receivers can achieve near-optimal performance, but complexity is still exponential in the number of users (for the multiuser detection applications) or the channel ISI length (for single user ISI applications) unless significant suboptimal modifications are made.

Honig et al. [36] developed a turbo receiver for synchronous CDMA that reduces complexity by using a suboptimal linear filtering operation for multiuser detection. Later, Wang and Poor developed a low-complexity turbo receiver for coded asynchronous CDMA in fading-multipath channels that relies on a MMSE-based multiuser detector working in conjunction with a MAP channel decoder [2]. Complexity was reduced to $O(N^2\tau^2 + 2^\nu)$ where $N$ is the processing gain and $\tau$ is the maximum delay in symbol intervals. In a separate work, Wang and Host-Madsen [3] developed (non-iterative) multiuser detectors for CDMA uplink environments in which the receiver has knowledge of the signature sequences of all of the users within the cell, but no knowledge of the sequences of users outside the cell. They termed these receivers “group-blind”. In this chapter we merge Wang and Poor’s low-complexity turbo receiver structure with the concept of group-blind multiuser detection to develop a turbo group-blind receiver for synchronous and asynchronous CDMA in which some of the users have spreading codes that are unknown. We will compare the performance of the new turbo group-blind receiver to the traditional turbo receiver developed in [2]. We will see that the group-blind turbo receiver provides a significant performance gain over a non-iterative receiver in this environment, while the traditional turbo receiver provides little performance gain through iteration when unknown users are present.

The remainder of this chapter is organized as follows. In Section 2, the system under study is described. In Section 3 we develop the soft-input soft-output (SISO)
MMSE group-blind multiuser detector that is used in our system. Simulation results are presented in Section 4, and Section 5 concludes.

B. System Description

We consider a coded synchronous CDMA system with $K$ users, employing normalized modulation waveforms $s_1, s_2, \ldots, s_K$, and signaling through an AWGN channel. A block diagram of the transmitter/receiver structure appears in Figure 5. The binary information bits for user $k$, $\{d_k(n)\}$, are encoded using a convolutional code, resulting in a coded bit stream $\{b_k(m)\}$. A code-bit interleaver is used to reduce the probability of error bursts and to remove correlation in the coded bit stream. The coded, interleaved bits are then mapped to BPSK symbols, yielding a symbol stream $\{b_k(i)\}$. Each symbol is then modulated by a spreading waveform $s_k$, and transmitted through the channel. The received signal is the superposition of the $K$ users’ transmitted signals plus the ambient noise, given by

$$
\mathbf{r}(i) = \sum_{k=1}^{K} A_k b_k(i) \mathbf{s}_k + \mathbf{n}(i) \tag{3.1}
$$

$$
= \mathbf{SAb}(i) + \mathbf{n}(i), \quad i = 0, \ldots, M - 1. \tag{3.2}
$$

In (3.2), $M$ is the number of data symbols per user per frame; $\mathbf{A} \triangleq \text{diag}(A_1, \ldots, A_K)$ where $A_k$ is the received amplitude of the $k$-th user; $\mathbf{b}(i) \triangleq [b_1(i) \cdots b_K(i)]^T$ where $b_k(i)$ denotes the $i$-th symbol of the $k$-th user; $\mathbf{S} \triangleq [\mathbf{s}_1 \cdots \mathbf{s}_K]$ where $\mathbf{s}_k$ is the normalized spreading waveform of the $k$-th user; $\mathbf{n}(i)$ is a zero mean i.i.d. Gaussian noise vector with variance $\sigma^2$ that is independent of the symbol sequences. The spreading waveform is of the form

$$
\mathbf{s}_k \triangleq \frac{1}{\sqrt{N}}[\beta_0^k \beta_1^k \cdots \beta_{N-1}^k]^T, \quad \beta_j^k \in \{+1, -1\}, \tag{3.3}
$$
where $N$ is the processing gain.

In the group-blind multiuser detection scenario, we assume we have knowledge of the first $\tilde{K} (\tilde{K} \leq K)$ users’ spreading sequences (and received amplitudes), whereas the rest of the users are unknown to the receiver. Denote $\hat{S}$ as the matrix consisting of the first $\tilde{K}$ columns of $S$. Denote the remaining $\hat{K} = K - \tilde{K}$ columns of $S$ by $\tilde{S}$. These signature sequences are unknown to the receiver. Let $\hat{b}(i)$ be the $\hat{K}$-vector containing the first $\hat{K}$ bits of $b(i)$ and let $\tilde{b}(i)$ contain the remaining $\tilde{K}$ bits. Similarly, denote $\hat{A} \overset{\Delta}{=} \text{diag}(A_1, \ldots, A_{\hat{K}})$ and $\tilde{A} \overset{\Delta}{=} \text{diag}(A_{\hat{K}+1}, \ldots, A_K)$. Then we may write (3.2) as

$$r(i) = \hat{S}\hat{A}\hat{b}(i) + \hat{S}\tilde{A}\tilde{b}(i) + n(i). \quad (3.4)$$

Since we do not have knowledge of $\tilde{S}$ we cannot hope to demodulate $\tilde{b}(i)$. We therefore write (3.4) as

$$r(i) = \hat{S}\hat{A}\hat{b}(i) + I(i) + n(i), \quad (3.5)$$

where $I(i) \overset{\Delta}{=} \hat{S}\tilde{A}\tilde{b}(i)$ is regarded as an interference term that is to be estimated and removed by our multiuser detector before it computes the $a$ posteriori log-likelihood ratios (LLRs) for the bits in $\tilde{b}(i)$.

In Figure 5, the MAP channel decoder accepts, as inputs, the set of extrinsic LLRs of all the code bits in the frame. It delivers, as output, updated LLRs of the code bits and, at the last iteration, the LLRs of the information bits. A forward-backward recursion of the type in [22] was developed in [2] and was used here. For brevity, the details are omitted. We note in passing that although some related work has been completed [37], the issue of passing extrinsic information versus full likelihood ratio information is not fully resolved. It may be true that for some system loads, passing the full likelihood ratio information may improve performance. Since a full analysis is beyond the scope of this letter, we will use the standard approach of
Fig. 5. Turbo group-blind multiuser detector transmitter/receiver structure.

passing extrinsic information.

C. SISO Group-Blind Multiuser Detectors

The heart of the turbo group-blind receiver is the soft-input soft-output (SISO) group-blind multiuser detector. The detector accepts, as inputs, the a priori LLRs for the code bits of the known users delivered by the SISO MAP channel decoder and produces, as outputs, updated LLRs for these code bits. This is accomplished by soft interference cancellation and MMSE filtering. Specifically, using the a priori LLRs and knowledge of the signature sequences and received amplitudes of the known users, the detector performs a soft-interference cancellation for each user, in which
estimates of the multiuser interference from the other known users and an estimate for the interference caused by the unknown users are subtracted from the received signal. This is in contrast to previously developed turbo multiuser detectors which ignore the interference from unknown users. Residual interference is suppressed by passing the resulting signal through an MMSE filter. The a posteriori LLR can be computed from the MMSE filter output. In this section we develop SISO group-blind detectors for synchronous and asynchronous CDMA.

1. SISO Group-Blind Detector for Synchronous CDMA

The detector first forms soft estimates of the user code bits as \( \tilde{b}_k(i) \triangleq E\{b_k(i)\} = \tanh\left(\frac{1}{2}\lambda_2[b_k(i)]\right) \) [2] where \( \lambda_2[b_k(i)] \) is the a priori LLR of the \( k \)-th bit during the \( i \)-th time slot delivered by the MAP decoder and is given by

\[
\lambda_2[b_k(i)] = \log \frac{\Pr[b_k(i) = +1]}{\Pr[b_k(i) = -1]}, \quad 1 \leq k \leq \tilde{K}, 0 \leq i \leq M - 1. \tag{3.6}
\]

We denote hard estimates of the code bits as \( \hat{b}_k(i) \triangleq \text{sign}[\tilde{b}_k(i)] \) and define

\[
\hat{b}(i) \triangleq [\hat{b}_1(i) \ \hat{b}_2(i) \cdots \hat{b}_K(i)]^T. \tag{3.7}
\]

In the next step we form an estimate of interference of the unknown users, \( I(i) \), which we denote by \( \hat{I}(i) \). We begin by forming a preliminary estimate

\[
\gamma(i) \triangleq r(i) - \tilde{S}\tilde{A}\hat{b}(i) \tag{3.8}
\]

\[
= \tilde{S}\tilde{A}\hat{b}(i) + \tilde{S}\tilde{A}\tilde{b}(i) + n(i) - \tilde{S}\tilde{A}\hat{b}(i) \tag{3.9}
\]

\[
= \tilde{S}\tilde{A}[\hat{b}(i) - \hat{b}(i)] + \tilde{S}\tilde{A}\hat{b}(i) + n(i) \tag{3.10}
\]

\[
= \tilde{S}\tilde{A}d(i) + \tilde{S}\tilde{A}\hat{b}(i) + n(i), \tag{3.11}
\]

where \( d(i) \triangleq [d_1(i) \ d_2(i) \cdots d_{\tilde{K}}(i)]^T \) and \( d_j(i), 1 \leq j \leq \tilde{K}, \) is a random variable
defined by
\[ d_j(i) \overset{\triangle}{=} b_j(i) - \hat{b}_j(i). \tag{3.12} \]

We will see that our ability to form a soft estimate for \( d_j(i) \) will allow us to perform the soft interference cancellation mentioned above. Clearly, \( d_j(i) \) can take on one of three values, \( \{-2, 0, 2\} \), i.e. 0 or \( 2b_j(i) \). The probability that \( d_j(i) \) is equal to zero is the probability that our hard estimate is correct and is given by
\[ \Pr[d_j(i) = 0] = \Pr \left[ b_j(i) = \text{sign} \left\{ \tanh \left( \frac{\lambda_2[b_j(i)]}{2} \right) \right\} \right]. \tag{3.13} \]

It is shown in [2] that for \( b \in \{ -1, 1 \} \), the probability that \( b_j(i) = b \) is given by
\[ \Pr[b_j(i) = b] = \frac{1}{2} + \frac{b}{2} \tanh \left( \frac{\lambda_2[b_j(i)]}{2} \right) \tag{3.14} \]

We substitute \( \text{sign} \left\{ \tanh \left( \frac{\lambda_2[b_j(i)]}{2} \right) \right\} \) for \( b \) in (3.14) and we find that
\[ \Pr[d_j(i) = 0] = \frac{1}{2} \left[ 1 + \text{sign} \left\{ \tanh \left( \frac{\lambda_2[b_j(i)]}{2} \right) \right\} \tanh \left( \frac{\lambda_2[b_j(i)]}{2} \right) \right] \tag{3.15} \]
\[ = \frac{1}{2} \left[ 1 + \tanh \left( \frac{|\lambda_2[b_j(i)]|}{2} \right) \right] \tag{3.16} \]

Therefore, \( d_j(i) \) is a random variable that can be described as
\[ d_j(i) = \begin{cases} 0 & \text{with probability } \frac{1}{2} + \frac{1}{2} \tanh \left( \frac{|\lambda_2[b_j(i)]|}{2} \right), \\ 2b_j(i) & \text{with probability } \frac{1}{2} - \frac{1}{2} \tanh \left( \frac{|\lambda_2[b_j(i)]|}{2} \right). \end{cases} \tag{3.17} \]

Denote by \( U_u \) the \( K \) largest eigenvectors of the eigendecomposition of \( E \{ \gamma(i)\gamma(i)^T \} \).

When perfect prior information is available, \( d(i) = 0 \) and \( U_u \) represents the (exact) signal subspace of the unknown users, i.e. the interference subspace. In order to refine our estimate of \( I(i) \) we project \( \gamma(i) \) onto \( U_u \). The result is
\[ \hat{I}(i) = U_u U_u^T \left\{ \tilde{S} \tilde{A}d(i) + \tilde{S} \tilde{A}\hat{b}(i) + n(i) \right\}. \tag{3.18} \]
If we denote \( \hat{S} \triangleq U_u U_u^T \hat{S} \) and \( \mathbf{v}(i) \triangleq U_u U_u^T \mathbf{n}(i) \), we have

\[
\hat{I}(i) = \hat{S} \hat{A} \mathbf{d}(i) + \hat{S} \hat{A} \hat{\mathbf{b}}(i) + \mathbf{v}(i).
\]  

(3.19)

Now we subtract the interference estimate from the received signal and form a new vector

\[
\zeta(i) \triangleq \mathbf{r}(i) - \hat{I}(i) = \hat{S} \hat{A} \hat{\mathbf{b}}(i) - \hat{S} \hat{A} \mathbf{d}(i) + \mathbf{w}(i),
\]  

(3.20)

(3.21)

where \( \mathbf{w}(i) \triangleq \mathbf{n}(i) - \mathbf{v}(i) \).

For each known user we perform a soft interference cancellation on \( \zeta(i) \) to obtain

\[
\mathbf{r}_k(i) \triangleq \zeta(i) - \hat{S} \hat{A} \hat{\mathbf{b}}_k(i) + \hat{S} \hat{A} \hat{\mathbf{d}}(i), \quad 1 \leq k \leq \tilde{K},
\]  

(3.22)

where \( \hat{\mathbf{b}}_k(i) \triangleq [\hat{b}_1(i) \cdots \hat{b}_{k-1}(i) \ 0 \ \hat{b}_{k+1}(i) \cdots \hat{b}_{\tilde{K}}(i)] \) and \( \hat{\mathbf{d}}(i) \triangleq [\hat{d}_1(i) \ \hat{d}_2(i) \cdots \hat{d}_{K}(i)]^T \) where \( \hat{d}_j(i) \) is a soft estimate for \( d_j(i) \) and is given by

\[
\hat{d}_j(i) \triangleq E \{ d_j(i) \} \quad (3.23)
\]

\[
= E \{ E \{ d_j(i) \mid b_j(i) \} \} \quad (3.24)
\]

\[
= \hat{b}_j(i) \left[ 1 - \tanh \left( \frac{|\lambda_2[b_j(i)]|}{2} \right) \right], \quad 1 \leq j \leq \tilde{K}. \quad (3.25)
\]

Substituting (3.21) into (3.22) we obtain

\[
\mathbf{r}_k(i) = \hat{S} \hat{A} [\hat{\mathbf{b}}(i) - \hat{\mathbf{b}}_k(i)] - \hat{S} \hat{A} [\mathbf{d}(i) - \hat{\mathbf{d}}(i)] + \mathbf{w}(i) \quad (3.26)
\]

\[
= \tilde{\mathbf{H}} [\hat{\mathbf{b}}(i) - \hat{\mathbf{b}}_k(i)] - \tilde{\mathbf{H}} [\mathbf{d}(i) - \hat{\mathbf{d}}(i)] + \mathbf{w}(i), \quad (3.27)
\]

where \( \tilde{\mathbf{H}} \triangleq \hat{S} \hat{A} \) and \( \hat{\mathbf{H}} \triangleq \hat{S} \hat{A} \).

An instantaneous linear MMSE filter is then applied to \( \mathbf{r}_k(i) \) to obtain

\[
z_k(i) \triangleq \mathbf{x}_k(i)^T \mathbf{r}_k(i).
\]  

(3.28)
The filter \( x_k(i) \in \mathbb{R}^N \) is chosen to minimize the mean-squared error between the code bit \( b_k(i) \) and the filter output \( z_k(i) \), i.e.,

\[
x_k(i) \overset{\Delta}{=} \arg \min_{x \in \mathbb{R}^N} E\{[b_k(i) - x^T r_k(i)]^2\},
\]

where the expectation is with respect to the ambient noise and the interfering users. The solution to (3.29) is given by

\[
x_k(i) = E\{r_k(i)r_k(i)^T\}^{-1}E\{b_k(i)r_k(i)\}.
\]

It is easy to show that

\[
E\{r_k(i)r_k(i)^T\} = \begin{bmatrix} \hat{H} & \tilde{H} \end{bmatrix} \begin{bmatrix} -\tilde{b}(i) - \tilde{b}_k(i) \\ d(i) - \tilde{d}(i) \end{bmatrix} \begin{bmatrix} -\tilde{b}(i) - \tilde{b}_k(i) \\ d(i) - \tilde{d}(i) \end{bmatrix}^T \begin{bmatrix} \tilde{H}^T \\ \tilde{H}^T \end{bmatrix} + \sigma^2 [I - U_u U_u^T] \]

\[
= \mathcal{H} \text{Cov} \left\{ \frac{\tilde{b}(i) - \tilde{b}_k(i)}{d(i) - \tilde{d}(i)} \right\} \mathcal{H}^T + \sigma^2 [I - U_u U_u^T], \quad (3.31)
\]

where \( \mathcal{H} \overset{\Delta}{=} [\hat{H} \; \tilde{H}] \). \( \Delta_k(i) \) has size \( 2\tilde{K} \times 2\tilde{K} \) and may be partitioned into four diagonal \( \tilde{K} \times \tilde{K} \) blocks as

\[
\Delta_k(i) = \begin{bmatrix} \Delta_{11}(i) & \Delta_{12}(i) \\ \Delta_{21}(i) & \Delta_{22}(i) \end{bmatrix},
\]

where, for convenience, we have dropped the user index \( k \) from the submatrix notation. The diagonal elements of \( \Delta_{11}(i) \) are given by

\[
[\Delta_{11}(i)]_{jj} = \text{Var}\{b_j(i)\} = \begin{cases} 1 - \tilde{b}_j^2(i) & 1 \leq j \leq \tilde{K}, j \neq k \\ 1 & j = k \end{cases}
\]

(3.35)
Similarly, the diagonal elements of $\mathbf{\Delta}_{22}(i)$ are given by

$$[\mathbf{\Delta}_{22}(i)]_{jj} = \text{Var}\{d_j(i)\} = 2\alpha_j(i) - \tilde{b}_j^2(i)\alpha_j^2(i), \quad 1 \leq j \leq K, \quad (3.36)$$

where

$$\alpha_j(i) \triangleq 1 - \tanh\left(\frac{|\lambda_2[b_j(i)]|}{2}\right). \quad (3.38)$$

The diagonal elements of $\mathbf{\Delta}_{12}(i)$ and $\mathbf{\Delta}_{21}(i)$ are identical and are given by

$$[\mathbf{\Delta}_{12}(i)]_{jj} = -\text{Cov}\{b_j(i), d_j(i)\} = \alpha_j(i)[\tilde{b}_j^2(i) - 1], \quad 1 \leq j \leq K. \quad (3.39)$$

It is also easy to see that

$$E\{b_k(i)r_k(i)\} = \bar{\mathbf{H}}\mathbf{e}_k - \alpha_k(i)\bar{\mathbf{H}}\mathbf{e}_k, \quad (3.41)$$

where $\mathbf{e}_k$ is a $K$-vector whose elements are all zero except for the $k$-th element which is 1. Using (3.32) and (3.41) in (3.30), we may write the MMSE filter for user $k$ as

$$\mathbf{x}_k(i) = [\mathbf{H}\mathbf{\Delta}_k(i)\mathbf{H}^T + \sigma^2[I - \mathbf{U}_k\mathbf{U}_k^T]]^{-1}\left[\bar{\mathbf{H}}\mathbf{e}_k - \alpha_k(i)\bar{\mathbf{H}}\mathbf{e}_k\right]. \quad (3.42)$$

If we make the common assumption that the MMSE filter output is Gaussian $[?]$, we may write

$$\mathbf{z}_k(i) \triangleq \mathbf{x}_k^T(i)\mathbf{r}_k(i) = \mu_k(i)b_k(i) + \eta_k(i), \quad (3.43)$$

where $\mu_k(i)$ is the equivalent amplitude of the $k$-th user’s signal at the filter output, and $\eta_k(i) \sim \mathcal{N}(0, \nu_k^2(i))$ is a Gaussian noise sample. We may compute the parameter
\[
\mu_k(i) = E\{z_k(i)b_k(i)\} = x_k^T E\{b_k(i)r_k(i)\} = \left[\bar{H}^T \Theta_k^{-1}(i) \bar{H}\right]_{kk} + \alpha_k^2(i) \left[\bar{H}^T \Theta_k^{-1}(i) \bar{H}\right]_{kk} - 2\alpha_k(i) \left[\bar{H}^T \Theta_k^{-1}(i) \bar{H}\right]_{kk}.
\]

where \( \Theta_k(i) = \left[\mathcal{H}\Delta_k(i) \mathcal{H}^T + \sigma^2 \left[I - U_u U_u^T\right]\right] \).

Finally, the extrinsic information, \( \lambda_1[b_k(i)] \), delivered by the SISO multiuser detector is given by

\[
\lambda_1[b_k(i)] \equiv \log \frac{p[z_k(i)|b_k(i) = +1]}{p[z_k(i)|b_k(i) = -1]} = -\frac{[z_k(i) - \mu_k(i)]^2}{2\nu_k^2(i)} + \frac{[z_k(i) + \mu_k(i)]^2}{2\nu_k^2(i)} = \frac{2z_k(i)}{1 - \mu_k(i)}.
\]

This algorithm is summarized on page 37 in Table II. Each step in the algorithm is annotated with its approximate complexity in floating point operations per user per symbol. The major computation involved in generating the MMSE filter output is the inversion of the matrix \( \Theta_k(i) \) in Step (5). Notice, however, that \( \mathcal{H}\Delta_k(i) \mathcal{H}^T = \bar{H} \Delta_{11}(i) \bar{H}^T + \bar{H} \Delta_{21}(i) \bar{H}^T + \bar{H} \Delta_{12}(i) \bar{H}^T + \bar{H} \Delta_{22}(i) \bar{H}^T \) can be written as the sum of \( 8K \) vector outer products (vector length \( N \)) and \( U_u U_u^T \) can be written as the sum of \( K \) vector outer products (vector length \( N \)) and is independent of \( k \). Hence, the matrix inversion can be computed iteratively using the matrix inversion lemma as in [2], resulting in a complexity of \( O \left(N^2 + \frac{KN^2}{MK}\right) \) rather than \( O(N^3) \).
2. Sliding Window Group-Blind Detector for Asynchronous CDMA

It is not difficult to extend the results of the previous subsection to asynchronous CDMA. The received signal due to user $k (1 \leq k \leq K)$ is given by

$$y_k(t) = A_k \sum_{i=0}^{M-1} b_k[i] \sum_{j=0}^{N-1} c_k[j] \psi(t - jT_c - iT - d_k), \quad (3.51)$$

where $d_k$ is the delay of the $k$-th user’s signal, $\{c_k[j]\}_{j=0}^{N-1}$ is a signature sequence of $\pm 1$’s assigned to the $k$-th user and $\psi(t)$ is a normalized chip waveform of duration $T_c = T/N$. The total received signal, given by

$$r(t) = \sum_{k=1}^{K} y_k(t) + v(t), \quad (3.52)$$

is matched filtered to the chip waveform and sampled at the chip rate, The $n$-th matched filter output during the $i$-th symbol interval is

$$r[i, n] \triangleq \begin{array}{c}
\int_{iT+nT_c}^{iT+(n+1)T_c} r(t) \psi(t - iT - nT_c) dt \\
= \sum_{k=1}^{K} \left\{ \int_{iT+nT_c}^{iT+(n+1)T_c} \psi(t - iT - nT_c) y_k(t) dt \right\} \\
+ \int_{iT+nT_c}^{iT+(n+1)T_c} v(t) \psi(t - iT - nT_c) dt.
\end{array} \quad (3.53)$$

Substituting (3.51) into (3.53) we obtain

$$y_k[i, n] = A_k \sum_{p=0}^{M-1} b_k[p] \sum_{j=0}^{N-1} c_k[j] \int_{iT+nT_c}^{iT+(n+1)T_c} \psi(t - iT - nT_c) \psi(t - jT_c - pT - d_k) dt \quad (3.54)$$

$$= \sum_{p=0}^{M-1} b_k[i - p] A_k \sum_{j=0}^{N-1} c_k[j] \int_{0}^{T_c} \psi(t) \psi(t - jT_c + nT_c + pT - d_k) dt \quad (3.55)$$
where \( \tau_k \triangleq 1 + \lceil (d_k + T_c)/T \rceil \). Then

\[
\begin{aligned}
    r[i, n] &= h_k[0, n]b_k[i] + \sum_{j=1}^{\tau_k-1} h_k[j, n]b_k[i-j] + \sum_{k' \neq k} y_{k'}[i, n] + v[i, n]. \\
\end{aligned}
\]  

(3.56)

Denote

\[
\begin{aligned}
    r[i_0] &\triangleq \begin{bmatrix} r[i, 0] \\ \vdots \\ r[i, N-1] \end{bmatrix}, \\
    v[i_0] &\triangleq \begin{bmatrix} v[i, 0] \\ \vdots \\ v[i, N-1] \end{bmatrix}, \\
    b[i_0] &\triangleq \begin{bmatrix} b_1[i] \\ \vdots \\ b_K[i] \end{bmatrix}.
\end{aligned}
\]  

(3.57)

and, for \( j = 0, 1, \ldots, \tau_k - 1 \),

\[
\begin{aligned}
    H[j] &\triangleq \begin{bmatrix} h_1[j, 0] & \cdots & h_K[j, 0] \\ \vdots & \ddots & \vdots \\ h_1[j, N-1] & \cdots & h_K[j, N-1] \end{bmatrix}.
\end{aligned}
\]  

(3.58)

Then

\[
\begin{aligned}
    r[i] &= H[i_0] \ast b[i_0] + v[i_0].
\end{aligned}
\]  

(3.59)

By stacking \( \triangleq \max_k \tau_k \) successive received sample vectors, we define

\[
\begin{aligned}
    \underline{r} &\triangleq \begin{bmatrix} r[i_0] \\ \vdots \\ r[i + \tau - 1] \end{bmatrix}, \\
    \underline{v} &\triangleq \begin{bmatrix} v[i_0] \\ \vdots \\ v[i + \tau - 1] \end{bmatrix}, \\
    \underline{b} &\triangleq \begin{bmatrix} b[i - \tau + 1] \\ \vdots \\ b[i + \tau - 1] \end{bmatrix}.
\end{aligned}
\]  

(3.60)

and

\[
\begin{aligned}
    \overline{H} &\triangleq \begin{bmatrix} H[\tau - 1] & \cdots & H[0] & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & H[\tau - 1] & \cdots & H[0] \end{bmatrix}.
\end{aligned}
\]  

(3.61)
where \( r \triangleq K(2\ell - 1) \). Then we can write the received signal in matrix form as

\[
\mathbf{r}[i] = \mathbf{Hb}[i] + \mathbf{v}[i].
\]  

(3.62)

Define the set of matrices \( \{\mathbf{H}_j\}_{j=0}^{2\ell-2} \) such that \( \mathbf{H}_j \) is the \( N\ell \times \bar{K} \) matrix composed of columns \( jK + 1 \) through \( jK + \bar{K} \) of the matrix \( \mathbf{H} \). We define the matrix \( \mathbf{\check{H}} \triangleq [\mathbf{\check{H}}_0 \ \mathbf{\check{H}}_1 \cdots \mathbf{\check{H}}_{2\ell-2}] \). The size of \( \mathbf{\check{H}} \) is \( N\ell \times \bar{K}(2\ell - 1) \). We denote by \( \mathbf{\hat{H}} \) the matrix that contains the remaining \( \bar{K}(2\ell - 1) \) columns of \( \mathbf{H} \). We define \( \hat{\mathbf{b}}[i] \) and \( \check{\mathbf{b}}[i] \) by performing a similar separation of the elements of \( \mathbf{b}[i] \). Then we may write (3.62) as

\[
\mathbf{r}[i] = \mathbf{\hat{H}\check{b}}[i] + \mathbf{\check{H}\hat{b}}[i] + \mathbf{v}[i].
\]  

(3.63)

This equation is the asynchronous analog to (3.4). We can obtain estimates of \( b_1[i], b_2[i], \ldots, b_{\bar{K}}[i] \) with straightforward modifications to the algorithm in Table II.

D. Simulation Results

In this section we present simulation results to demonstrate the performance of the proposed turbo group-blind multiuser receiver for asynchronous CDMA. The processing gain of the system is 7 and the total number of users is 7. The number of known users is either 2 or 5, as noted on the figures. The spreading sequences are random and the same sequences are used for all simulations. All users employ the same rate \( \frac{1}{2} \), constraint length 3 convolutional code (with generators \( g_1 = [110] \) and \( g_2 = [111] \)). Each user uses a different random interleaver, and the same interleavers are used in all simulations. The block size of information bits for each user is 128. The maximum delay, in symbol intervals is 1. All users use the same transmitted power and the chip pulse waveform is a raised cosine with roll-off factor .5.
Figure 6 illustrates the average bit-error-rate performance of the known users for the group-blind turbo receiver and the conventional turbo receiver [2] for the first 4 iterations. The number of known users is 5. For the sake of comparison, we have also included plots for the conventional turbo receiver when all of the users are known. The three sets of plots in this figure are denoted in the legend by “GB-MUD”, “TMUD”, and “ALL KNOWN” respectively. Note that the curves for the first iteration are identical for GBMUD and TMUD. Hence we have suppress the plot of the first iteration for TMUD to improve clarity. Notice that iteration does not significantly improve the performance of the conventional turbo receiver while the group-blind receiver provides significant gains through iteration at moderate and high signal-to-noise ratios. We can also see that the use of more than three iterations does not provide significant benefits.

In Figure 7, the number of known users has been changed to 2. As we would expect, there is a performance degradation for both the conventional and group-blind turbo receivers. In fact, the conventional receiver gains nothing through iteration for this scenario because there are now 5 users whose interference is simply ignored. It is also apparent that the group-blind turbo receiver will not be able to mitigate all of the interference of the unknown users, even for a large number of iterations. This is due, in part, to the use of an imperfect interference subspace estimate in the SISO group-blind multiuser detector.

E. Conclusions

In this chapter we have developed a new turbo multiuser receiver for CDMA systems that is capable of suppressing interference not only from known users, but also from users whose signature sequences and received amplitudes are unknown. This
Fig. 6. Performance of the group-blind iterative multiuser receiver with 5 known users. Curves denoted GB-TMUD are produced using the turbo group-blind multiuser receiver and those denoted TMUD are produced using the traditional turbo multiuser receiver. Also included are plots for TMUD when all users are known.
Fig. 7. Performance of the group-blind iterative multiuser receiver with 2 known users. Curves denoted GB-TMUD are produced using the turbo group-blind multiuser receiver and those denoted TMUD are produced using the traditional turbo multiuser receiver. Also included are plots for TMUD when all users are known.
technique differs from previously developed turbo multiuser receivers that ignore unknown interferers. We have seen that this so-called group-blind turbo multiuser receiver provides a significant performance improvement over a non-iterative receiver, whose performance is illustrated in Figures 6 and 7 by the first iteration. We have also seen that the traditional turbo receiver fails to provide a useful performance gain over a non-iterative receiver when unknown users are present. The new algorithm is summarized in Table II.
Table II

<table>
<thead>
<tr>
<th>SISO GROUP-BLIND MULTIUSER DETECTION ALGORITHM.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given: $r(l), \lambda_2[b_j(l)], \ 0 \leq l \leq M - 1, 1 \leq j \leq \tilde{K}$.</td>
</tr>
<tr>
<td>1. $[O(1)]$ For $1 \leq k \leq \tilde{K}$ and for $0 \leq j \leq M - 1$, form soft and hard estimates of code bits: $\hat{b}_k(j) = \tanh[\lambda_2[b_k(j)]/2]$, $\hat{b}_k(j) = \text{sign}[	ilde{b}_k(j)]$. Define the vectors $\hat{b}(j) \triangleq [\hat{b}_1(j) \ \hat{b}_2(j) \cdots \hat{b}_K(j)]^T$ and $\tilde{b}<em>k(j) \triangleq [\tilde{b}<em>1(j) \cdots \tilde{b}</em>{k-1}(j) \ 0 \ \tilde{b}</em>{k+1}(j) \cdots \tilde{b}_K(j)]$.</td>
</tr>
<tr>
<td>2. Form $\hat{U}_u$, an estimate of $U_u$, by using the $K$ largest eigenvectors of $1/M \Gamma \Gamma^T$, where $\Gamma \triangleq [\gamma(0) \cdots \gamma(M - 1)]$ and $\gamma(j) \triangleq r(j) - \tilde{S} \tilde{A} \tilde{b}(j)$.</td>
</tr>
<tr>
<td>3. for $i = 0, 1, \ldots, M - 1$</td>
</tr>
<tr>
<td>(1). $[O\left(N^2/K\right)]$ Refine the estimate from step 2 by projection: $\hat{I}(i) = \hat{U}_u \hat{U}_u^T \gamma(i)$.</td>
</tr>
<tr>
<td>(2). $[O(1)]$ Define $\tilde{d}(i) \triangleq [\tilde{d}_1(i) \ \tilde{d}_2(i) \cdots \tilde{d}_K(i)]^T$ and compute $\tilde{d}_j(i)$ according to $\tilde{d}_j(i) = \tilde{b}_j(i)\alpha_j(i)$, where $\alpha_j(i)$ is defined in (3.38).</td>
</tr>
<tr>
<td>(3). $[O\left(N^2/K\right)]$ Subtract $\hat{I}(i)$ from $r(i)$ and for $1 \leq k \leq \tilde{K}$ perform soft interference cancellation: $r_k(i) = r(i) - \hat{I}(i) - \tilde{S} \tilde{A} \hat{b}_k(i) + \tilde{S} \tilde{A} \tilde{d}(i)$.</td>
</tr>
<tr>
<td>(4). $[O(1)]$ Calculate $\Delta_k(i)$ according to (3.34)-(3.40).</td>
</tr>
</tbody>
</table>
| (5). $O\left[N^2 + \tilde{K}N^2/MK\right]$ Calculate and apply the MMSE filter: $\begin{align*}
x_k(i) &= \begin{bmatrix} H \Delta_k(i) \mathcal{H}^T + \sigma^2 \left[I - \hat{U}_u \hat{U}_u^T \right] -1 \left[ \tilde{H} \epsilon_k - \alpha_k(i) \tilde{H} \epsilon_k \right] \\
z_k(i) &= x_k(i)^T r_k(i)
\end{align*}$

where $\mathcal{H} \triangleq [\tilde{H} \ \tilde{H}]$ and where $\tilde{H} \triangleq \tilde{S} \tilde{A}$ and $\tilde{H} \triangleq \tilde{S} \tilde{A}$. |
| (6). $[O(1)]$ Compute $\mu_k(i)$ according to (3.47). |
| (7). $[O(1)]$ For $1 \leq k \leq \tilde{K}$ compute the a posteriori LLR for code bit $b_k(i)$ via $\lambda_1[b_k(i)] = 2z_k(i)/[1 - \mu_k(i)]$. |
| end |
CHAPTER IV

LOW-COMPLEXITY BLIND ADAPTIVE MULTIUSER DETECTION

A. Introduction

Blind multiuser detection using subspace techniques was first developed in depth by Poor and Wang [10, 11]. Such techniques were appropriate for downlink environments where only the desired users’ code is available. More recently, these subspace techniques were extended by Wang and Host-Madsen to uplink environments where the base station knows the codes of in-cell users, but not those of users outside the cell [3]. This new family of detectors has been termed group-blind multiuser detectors. One attractive member of this family, the group-blind linear hybrid detector, performs very well compared with the other group-blind detectors, even though it has the lowest computational complexity. In this chapter we develop a new, low-complexity, high performance subspace tracking algorithm and use it, along with the closed form expression for the hybrid group-blind detector, to develop an adaptive group-blind multiuser detector for slowly-varying asynchronous dispersive CDMA channels. We will also compare the performance of the group-blind detector to that of the blind detector that makes use only of the composite waveform of the user of interest.

The rest of this chapter is organized as follows. In Section 2, we summarize the signal model. In Section 3, we review subspace methods of blind and group-blind multiuser detection In Section 4 we address the role of subspace tracking in our adaptive receiver and we introduce a new low-complexity, high performance subspace tracking algorithm. Simulation results are provided in Section 5 and Section 6 concludes.
B. Signal Model

Consider a K-user binary communication system, employing normalized modulating wave-forms $s_1, s_2, \ldots, s_k$, and signaling through their respective multipath channels with additive Gaussian noise. The transmitted signal due to the $k$-th user is given by

$$x_k(t) = A_k \sum_{i=0}^{M-1} b_k[i] s_k(t - iT - d_k)$$  \hspace{1cm} (4.1)

where $M$ denotes the length of the data frame and $T$ denotes the information symbol interval; $A_k, \{b_k[i]\}$, and $d_k \in [0, T)$ denote respectively the amplitude, symbol stream, and the delay of the $k$-th user’s signal. We assume that for each $k$ the symbol stream, $\{b_k[i]\}$, is a collection of independent random variables that take on values of $+1$ and $-1$ with equal probability. Furthermore, we assume that the symbol streams of different users are independent. For the direct-sequence spread-spectrum (DS-SS) format, the user signaling waveforms have the form

$$s_k(t) = \sum_{j=0}^{N-1} c_k[j] \psi(t - jT_c), \quad 0 \leq t \leq T,$$  \hspace{1cm} (4.2)

where $N$ is the processing gain, $\{c_k[j]\}$ is a signature sequence of $\pm 1$’s assigned to the $k$-th user, and $\psi(t)$ is a normalized chip waveform of duration $T_c = T/N$. The $k$-th user’s signal, $x_k(t)$ propagates through a multipath channel whose impulse response is given by

$$g_k(t) = \sum_{l=1}^{L} \alpha_{kl} \delta(t - \tau_{kl}),$$  \hspace{1cm} (4.3)

where $L$ is the number of paths in the channel; $\alpha_{kl}$ and $\tau_{kl}$ are respectively the complex path gain and delay of the $l$-th path of the $k$-th user. It is assumed that the channel is slowly varying, so that the path gains and delays remain constant over the duration of one signal frame ($MT$). The received signal component due to the transmission of
the $k$-th user’s signal through the channel $g_k(t)$ is given by

$$y_k(t) = x_k(t) * g_k(t) = \sum_{i=0}^{M-1} b_k[i] \left[ A_k s_k(t - iT - d_k) * g_k(t) \right]$$

(4.4)

where

$$h_k(t) \triangleq A_k s_k(t - d_k) * g_k(t) = \sum_{j=0}^{N-1} c_k[j] \left[ A_k \sum_{l=1}^{L} \alpha_{kl} \psi(t - jT_c - d_k - \tau_{kl}) \right]$$

(4.5)

In (4.5), $\overline{g}_k(t)$ is the composite channel response, taking into account the effects of transmitter power, chip pulse waveform, and the multipath channel, given by

$$\overline{g}_k(t) \triangleq A_k \sum_{l=1}^{L} \alpha_{kl} \psi(t - d_k - \tau_{kl}).$$

(4.6)

Since $\psi(t)$ is zero outside the interval $[0, T_c]$, $\overline{g}_k(t)$ is zero outside the interval $[d_k + \tau_{k1}, d_k + \tau_{kL} + T_c]$. Hence, the composite signature waveform $h_k(t)$ of the $k$-th user is zero outside the interval $[d_k + \tau_{k1}, d_k + \tau_{kL} + T]$.

The total received signal at the base station receiver is the superposition of the $K$ user’s signals, plus additive Gaussian noise and is given by

$$r(t) = \sum_{k=1}^{K} y_k(t) + v(t),$$

(4.7)

where $v(t)$ is a zero mean complex Gaussian noise process. At the receiver, the received signal is matched filtered (to the chip waveform) and sampled at a multiple ($p$) of the chip rate, i.e., the sampling interval is $\Delta = T_c/p = T/P$ where $P \triangleq pN$ is the total number of samples per symbol interval. The $n$-th received signal sample during the $i$-th symbol is given by

$$r[i, n] \triangleq r((iP + n)\Delta) = r(iT + n\Delta) = \sum_{k=1}^{K} y_k[i, n] + v[i, n].$$

(4.8)
Let \( \ell_k \triangleq \left\lfloor \frac{d_k + r_k T_s}{T} \right\rfloor \). Then using (4.4), we have
\[
y_k[i, n] \triangleq y_k(iT + n\Delta) = \sum_{j=0}^{M-1} b_k[j] h_k(iT + n\Delta - jT)
\]
\[
= \sum_{j=1}^{\ell_k} b_k[j] h_k(iT + n\Delta - jT) = \sum_{j=0}^{\ell_k} h_k[j, n] b_k[i - j], \tag{4.9}
\]
where (4.9) follows from the fact that \( h_k(t) \) is zero outside the interval \([0, (\ell_k + 1)T]\).

Hence, for the \( k \)-th user, (4.8) can be written as
\[
r[i, n] = h_k[0, n] b_k[i] + \sum_{j=1}^{\ell_k} h_k[j, n] b_k[i - j] + \sum_{j\neq k} y_{k'}[i, n] + v[i, n]. \tag{4.10}
\]

In (4.10), the first term contains the \( i \)-th bit of the \( k \)-th user; the second term contains the intersymbol interference (ISI) from the previous bits of the \( k \)-th user; the third term contains the multiple-access interference (MAI) from the other users; and the last term is the ambient channel noise. Denote
\[
\begin{align*}
\mathbf{r}[i] & \triangleq \begin{bmatrix}
  r[i, 0] \\
  \vdots \\
  r[i, P - 1]
\end{bmatrix}_{P \times 1},
\end{align*}
\]
\[
\begin{align*}
\mathbf{v}[i] & \triangleq \begin{bmatrix}
  v[i, 0] \\
  \vdots \\
  v[i, P - 1]
\end{bmatrix}_{P \times 1},
\end{align*}
\]
\[
\begin{align*}
\mathbf{b}[i] & \triangleq \begin{bmatrix}
  b_1[i] \\
  \vdots \\
  b_K[i]
\end{bmatrix}_{K \times 1},
\end{align*}
\]
\[
\begin{align*}
\mathbf{H}[j] & \triangleq \begin{bmatrix}
  h_1[j, 0] & \cdots & h_K[j, 0] \\
  \vdots & \ddots & \vdots \\
  h_1[j, P - 1] & \cdots & h_K[j, P - 1]
\end{bmatrix}_{P \times K},
\end{align*}
\tag{4.11}
\]

Then from (4.8) and (4.9) we have
\[
r[i] = \mathbf{H}[i] \mathbf{b}[i] + \mathbf{v}[i]. \tag{4.13}
\]

Define \( \ell \triangleq \max_{1 \leq k \leq K} \{\ell_k\} \). By stacking \( m \) successive received sample vectors, we
create the following quantities:

\[ \mathbf{r}[i] \triangleq \begin{bmatrix} r[i] \\ \vdots \\ r[i + m - 1] \end{bmatrix}_{p \times 1}, \quad \mathbf{v}[i] \triangleq \begin{bmatrix} v[i] \\ \vdots \\ v[i + m - 1] \end{bmatrix}_{p \times 1}, \quad \mathbf{b}[i] \triangleq \begin{bmatrix} b[i - 1] \\ \vdots \\ b[i + m - 1] \end{bmatrix}_{r \times 1} \]

\[ \mathbf{H} \triangleq \begin{bmatrix} H[i] & \cdots & H[0] & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & H[i] & \cdots & H[0] \end{bmatrix}_{p \times r} \]

(4.14)

where \( r \triangleq K(m + i). \) We can write (4.13) in matrix form as

\[ \mathbf{r}[i] = \mathbf{H}\mathbf{b}[i] + \mathbf{v}[i]. \]

(4.16)

The smoothing factor, \( m, \) is chosen such that \( m \geq \left\lceil \frac{m + K}{r - K} \right\rceil \) \( i \) for channel identifiability [3]. Note that the columns of \( \mathbf{H} \) (the composite signature vectors) contain information about both the timings and the complex path gains of the multipath channel of each user. Hence an estimate of these waveforms eliminates the need for separate estimates of the timing information \( \{\tau_{kl}\}_{i=1}^L. \)

C. Group-Blind Multiuser Detection

Since the ambient noise is white, i.e., \( \mathbb{E}\{\mathbf{v}[i]\mathbf{v}[i]^H\} = \sigma^2 \mathbf{I}, \) the autocorrelation matrix of the received signal in (4.16) is

\[ \mathbf{X} \triangleq \mathbb{E}\{\mathbf{r}[i]\mathbf{r}[i]^H\} = \mathbf{HH}^H + \sigma^2 \mathbf{I} \]

(4.17)

\[ = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \sigma^2 \mathbf{U}_n \mathbf{U}_n^H, \]

(4.18)

where (6.18) is the eigendecomposition of \( \mathbf{X}. \) In the group-blind multiuser detection scenario, we assume we have knowledge of the first \( \tilde{K}, \tilde{K} \leq K \) users’ spreading
sequences, whereas the rest of the users are unknown to the receiver.

Define the set of matrices \( \{ \tilde{H}_j \}_{j=0}^{m+n} \) such that \( \tilde{H}_j \) is the \( Pm \times \tilde{K} \) matrix composed of columns \( j\tilde{K} + 1 \) through \( j\tilde{K} + \tilde{K} \) of the matrix \( H \). We define the matrix \( \tilde{H} \triangleq [\tilde{H}_0 \ \tilde{H}_1 \cdots \tilde{H}_{m+n}] \). The size of \( \tilde{H} \) is \( Pm \times \tilde{r} \) where \( \tilde{r} \triangleq \tilde{K}(m+\ell) \). Then the group-blind linear hybrid detector for user \( k, k = 1, \ldots, \tilde{K} \) is given by the solution to the following constrained optimization problem:

\[
\mathbf{w}_k = \arg \min_{\mathbf{w} \in \text{range}(\tilde{H})} E \left\{ |b_k[i] - \mathbf{w}^H \mathbf{r}[i]|^2 \right\},
\]

subject to the constraint that \( \mathbf{w}^H \tilde{H} = \mathbf{1}_K^T \tilde{K}_{\ell+k} \) where \( \mathbf{1}_l \) is the vector of length \( \tilde{K}(m+\ell) \) each of whose elements is zero except the \( l \)-th element which is 1. Heuristically speaking, this detector zero-forces the interference caused by the \( \tilde{K} \) known users, and suppresses the interference from unknown users according to the MMSE criterion. The solution (form II) and the corresponding bit estimate for user \( k \) may be written as [3]

\[
\mathbf{w}_k = \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \tilde{H} \mathbf{H}^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \tilde{H} \mathbf{1}_{\tilde{K}_{k+\ell}} \quad (4.20)
\]

\[
\hat{b}_k[i] = \text{sgn} \left\{ \text{Re} \left( \mathbf{w}_k^H \mathbf{r}[i] \right) \right\}, \quad k = 1, 2, \ldots, \tilde{K}. \quad (4.21)
\]

The linear blind MMSE multiuser detector, which assumes knowledge only of the signature waveform of the user of interest is given by [10]

\[
\mathbf{m}_k = \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \tilde{H} \mathbf{1}_{\tilde{K}_{k+\ell}}. \quad (4.22)
\]

Of course, we require knowledge of the composite signature waveforms of the first \( \tilde{K} \) users in order to construct \( \tilde{H} \). To estimate these waveforms we take advantage of the fact that the noise subspace is orthogonal to the columnspace of \( \mathbf{H} \). However, computing the noise subspace increases complexity, so we adopt the blind, sequential
channel estimation technique developed by Poor and Wang in [38]. It produces estimates of the composite signature waveforms without the need of a noise subspace estimate. Note that there is an arbitrary phase ambiguity in the estimated channel state, which necessitates differential encoding and decoding of the transmitted data.

D. Adaptation via Subspace Tracking

Since the form-II hybrid group-blind detector may be written in closed form as a function of the signal subspace components, one may use a suitable subspace tracking algorithm in conjunction with this detector and a channel estimator to form an adaptive detector that is able to track changes in the number of users and their composite signature waveforms. Figure 8 contains a block diagram of such a receiver. The received signal $r[i]$ is fed into a subspace tracker which sequentially estimates the signal subspace components $(U_s, A_s)$. These estimates, along with the received signal, are fed to the blind sequential Kalman channel estimator developed in [38]. The linear hybrid group-blind detector (or the linear blind MMSE detector) is then constructed from the channel state estimate and the signal subspace component estimates.

Subspace trackers of various complexities and performance characteristics have appeared in the literature. In particular, the NA-CSVD [39] tracking algorithm was used successfully by Yu and Høst-Madsen for subspace tracking for group-blind MUD over synchronous CDMA channels [40].

1. QR-Jacobi Methods

QR-Jacobi methods constitute a family of SVD-based subspace tracking algorithms that rely extensively on Givens rotations during the updating process. This reduces complexity and has the advantage of maintaining the orthonormality of matrices.
Fig. 8. Adaptive receiver structure.

Members of this family include NA-CSVD, RO-FST [41], NASVD [42], and the algorithm developed by Moonen et. al. in [43].

Let $R(l) = [r[1] \cdots r[l]]$ denote a $N \times l$ matrix whose columns contain the first $l$ snapshots of the received signal. Define the matrix $\Gamma(l) = \text{diag}(\sqrt{\gamma}^{l-1}, \ldots, \sqrt{\gamma}, 1)$. Then the exponentially windowed sample correlation matrix is given by

$$C(l) = \left( \frac{1}{M(l)} \right) R(l) \Gamma(l) \cdot [R(l) \Gamma(l)]^H \quad (4.23)$$

where $M(l) = (1 - \gamma^l)/(1 - \gamma)$ is the effective window length. Generally speaking, SVD-based subspace tracking algorithms attempt to track the SVD of a data matrix of growing dimension, defined recursively as

$$\Gamma(l) R^H(l) = \begin{bmatrix} \sqrt{\gamma} \Gamma(l-1) R^H(l-1) \\ r^H[l] \end{bmatrix} \quad (4.24)$$
We may write the SVD of this matrix as

\[
\Gamma(l) R^H(l) = U(l) \Sigma(l) V^H(l) \quad (4.25)
\]

\[
= U(l) \begin{bmatrix} \Sigma_s(l) & 0 \\ 0 & \Sigma_n(l) \end{bmatrix} \begin{bmatrix} V_s^H(l) \\ V_n^H(l) \end{bmatrix} \quad (4.26)
\]

where \( V_s(l) \) is a matrix whose columns are eigenvectors that span the signal subspace and \( \text{diag}[\Sigma_s(l)] \) contains the square-root of the corresponding eigenvalues. The matrix \( U(l) \) need not be tracked. Furthermore, since the noise subspace does not need to be calculated for the algorithm used in this chapter, we do not need to track \( V_n(l) \) or \( \Sigma_n(l) \). This allows us to reduce complexity using noise averaging [44]. Since calculating the SVD from scratch at each iteration is time consuming and expensive, the issue then is how best to use the new measurement vector, \( r[l+1] \), to update the decomposition in (4.26).

Noise-averaged QR-Jacobi algorithms begin with a Householder transformation that rotates the noise eigenvectors such that the projection of the new measurement vector \( r[l+1] \) onto the noise subspace is parallel to the first noise vector, which we denote by \( v_n \). Specifically, let

\[
\begin{align*}
\mathbf{r}_s &= V_s(l)^H r[l+1] \\
\mathbf{v}_n &= \frac{r[l+1] - V_s(l)\mathbf{r}_s}{\beta}.
\end{align*}
\] (4.27)  (4.28)

where \( \beta = \|r[l+1] - V_s(l)\mathbf{r}_s\| \). Then we may write the modified factorization

\[
\begin{bmatrix} \sqrt{\gamma} \Gamma(l) R^H(l) \\ r^H[l+1] \end{bmatrix} = \begin{bmatrix} U(l) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{\gamma} \Sigma(l) \\ r^H \beta \end{bmatrix} \cdot [V_s(l)|\mathbf{v}_n|V_n^\perp]^H \quad (4.29)
\]

where \( V_n^\perp \) represents the subspace of \( V_n(l) \) that is orthogonal to \( \mathbf{v}_n \). The second step in QR-Jacobi methods, sometimes called the QR step, involves the use of Givens
rotations to zero each entry of the measurement vector’s projection on the signal subspace. We refer the reader to [45] for details concerning the use of Givens matrices for this purpose. The QR step replaces the last row in the middle matrix in the decomposition in (4.29) with zeros. These are row-type transformations involving premultiplication of the middle matrix with a sequence of orthogonal matrices. We do not need to accumulate these transformations in $U(l)$ since $U(l)$ does not need to be tracked.

The next step, diagonalization step, involves at least one set each of column-type and row-type rotations to further concentrate the energy in the middle matrix along its diagonal. Sometimes called the refinement step, this is where many of the existing algorithms begin to diverge. The RO-FST algorithm, for example, performs two fixed sets of rotations in the diagonalization step but leaves the middle matrix in upper triangular form and does not attempt a diagonalization. This is particularly efficient for applications that do not require a full set of eigenvalues, but is not useful here since the signal subspace eigenvalues are required for the construction of the detector. The NA-CSVD algorithm, on the other hand, attempts to optimize the choice of rotations to achieve the best diagonalization possible. The algorithm we present in the next section achieves near-optimal results (for noise-averaged trackers) while incurring a computational burden that is smaller than that of NA-CSVD.

2. NAHJ-FST Subspace Tracking

The algorithm we present here is a member of the QR-Jacobi family in the sense that it uses Givens rotations during the updating process. However, this algorithm avoids the QR step entirely. Instead of working with the SVD-type decomposition in (4.25),
we work with a decomposition of the form

$$C'(l) = V(l) \Sigma^2(l) V^H(l)$$  \hspace{1cm} (4.30)

where $C'(l) = M(l)C(l)$ and $\Sigma^2(l)$ is Hermitian and almost diagonal. This is simply an eigendecomposition except that we have relaxed the assumption that $\Sigma^2(l)$ is perfectly diagonal. At each iteration we use a Householder transformation and a vector outer product to update $\Sigma^2(l)$ directly. We then use single set of two-sided Givens rotations to partially diagonalize the resulting Hermitian matrix. There is no need for a separate QR-step. Essentially, the diagonalization process used in this algorithm is a partial implementation of the well known symmetric Jacobi SVD algorithm [45] (not to be confused with the family of QR-Jacobi update algorithms). This algorithm is used to find the eigenstructure of a general fixed symmetric matrix and is known to generate more accurate eigenvalues and eigenvectors than the symmetric QR SVD algorithm, but with a higher computational complexity [46]. However, we do not perform the full sweep of $r(r - 1)/2$ rotations required for the symmetric Jacobi algorithm, but only a carefully selected set of about $r$ rotations. This is sufficient because the matrix that we wish to diagonalize already has much of its energy concentrated along the diagonal. This is a situation that the Jacobi algorithm can take advantage of but which the QR algorithm cannot. The Jacobi algorithm also has an inherent parallelism which the QR algorithm does not. Table III, on page 55, contains a summary of this algorithm, which we term NAHJ-FST for Noise-Averaged Hermitian-Jacobi Fast Subspace Tracking.

a. The Algorithm

The first step in NAHJ-FST is the Householder transformation mentioned previously. The second step involves generating a modified factorization that maintains the equal-
ity $V(l)\Sigma^2(l)V^H(l) = \gamma V(l-1)\Sigma^2(l-1)V^H(l-1) + r(l)r^H(l)$. Step 3 requires that we apply $r + 1$ Givens rotations in order to partially diagonalize $R_s$. Ideally, we would apply these rotations to the off-diagonal elements that have the largest magnitude. However, since the off-diagonal maxima can be located anywhere in $R_s$, finding the optimal set of rotations requires an $O(r^2)$ search for each rotation. This leads to an $O(r^3)$ complexity algorithm. In order to maintain low complexity we have implemented a suboptimal alternative that is simple yet effective. Let $z = [r_s^H|\beta]^H$ be the vector whose outer product is used in the modified factorization of step 2. Suppose $i_0, 1 \leq i_0 \leq r + 2$ is the index of the element in $z$ that has the largest magnitude. The set of elements we choose to annihilate with the Givens rotations is given by $\{(R_s)_{i_0,j}\}_{j=1}^{r+2}, j \neq i_0$ where $(R_s)_{i_0,j}$ represents the element on the $i_0$-th row and $j$-th column of $R_s$. Of course if $(R_s)_{i_0,j}$ is annihilated, so is $(R_s)_{j,i_0}$. This choice of rotations is not optimal; in fact, since we retain the off-diagonal information from the previous iteration we cannot even be sure we annihilate the off-diagonal element in $R_s$ with the largest magnitude. Nevertheless, we see that the technique is very simple and is somewhat heuristically pleasing. Ultimately, performance is the measure of merit and simulations show that it performs very well.

In order to adapt to changes in the size of the signal subspace (number of users) our tracking algorithm must be rank-adaptive. We have adopted the Akaike information criterion method. This method is often used in subspace tracking algorithms and is documented in [47]. In order to use this algorithm we must track at least one extra eigenvalue/eigenvector pair. Hence the appearance of $r + 1$ in Table III.

b. Complexity

Complexity is a critical issue when considering subspace trackers for multiuser detection. Existing algorithms vary in complexity from $O(Nr)$ to $O(N^3)$. The NAHJ-FST
algorithm has complexity $O(Nr)$. In fact, when we consider trackers that deliver a complete set of signal subspace eigenvalues at each iteration, NAHJ-FST seems to have the lowest complexity of any algorithm that delivers comparable performance. Its nearest competitor appears to be NA-CSVD which has a complexity of $10Nr + 3N + 7.5r^2 + 7r$ floating point operations (flops) per iteration using real data. NAHJ-FST requires approximately $3r^2$ fewer flops iteration. For the parameters used in our simulations, this results in about 17% fewer flops per iteration.

It is not the aim of this chapter to present an exhaustive comparison of all of these QR Jacobi-type algorithms. It is sufficient to observe that NAHJ-FST has the lowest complexity of any algorithm that has been used for similar purposes, and that for the particular tracking problem we are addressing, the performance of the algorithm is close to the upper bound on the performance of all algorithms of this type. This upper bound is, at each iteration, given by an exact $O(N^3)$ rank-one SVD update of the entire noise and signal subspaces. We present more details in the next section.

E. Simulation Results

In this section we investigate the performance of our adaptive receiver in an asynchronous CDMA system. The processing gain $N = 15$ and the spreading codes are Gold codes of length 15. The chip pulse waveform is a raised cosine pulse with a roll-off factor of .5. The initial delay $d_k$ of each user is uniform on $[0, 4T_c]$. Each user’s channel has $L = 3$ paths. The delay of each path $\tau_{k,l}$ is uniform on $[0, 6T_c]$. Hence, the maximum delay spread is one symbol interval, i.e., $\tau = 1$. The fading gain of each path in each user’s channel is generated from a complex Gaussian distribution and is fixed for all simulations. The path gains in each user’s channel are normalized so
that each user’s signal arrives at the receiver with the same power. The oversampling
factor is \( p = 2 \) and the smoothing factor is \( m = 2 \). Hence, this system can accom-
modate up to 10 users. The forgetting factor for the subspace tracking algorithms is .995.

The performance measures are bit-error probability and the signal-to-interference
ratio defined by \( \text{SIR} \overset{\Delta}{=} E^2 \{ w^H r \} / \text{Var} \{ w^H r \} \), where the expectation is with respect
to the data bits of interfering users, the ISI bits, and the ambient noise. In the
simulations, the expectation operation is replaced by the time averaging operation.
SIR is a particularly useful figure of merit for MMSE detectors since it has been shown
[21] that the output of an MMSE detector is approximately Gaussian distributed.
Hence, the SIR values translate directly and simply to bit-error probabilities.

Figure 9 is a comparison of the adaptive performance of the MMSE blind de-
tector and the hybrid group-blind detector using the NAHJ-FST subspace tracking
algorithm. The SNR is fixed at 20dB. During the first 1000 iterations there are 8 total
users, 6 of which are known by the group-blind detector. At iteration 1000, 2 new
users are added to the system. At iteration 2000, one additional known user is added
and three unknown users vanish. We see that there is a substantial performance gain
using the group-blind detector at each stage and that convergence occurs in less than
500 iterations.

Figure 10 is created with parameters identical to Figure 9 except that the tracking
algorithm used is an exact rank-one SVD update. Again we see a significant improve-
ment in performance using the group-blind detector. More importantly, when we
compare Figures 9 and 10 we see very little difference between the performance we
obtain using NAHJ-FST and that we obtain using an exact SVD update.

Figure 11 represents the steady state bit-error-rate (BER) performance of our
receiver using NAHJ-FST and the exact SVD update for both blind and group blind
multiuser detection. The number of users is 8 and the number of known users is 6. At signal to noise ratios above about 11dB we see that the group-blind detectors provide a substantial improvement in BER. At lower SNR, the group-blind detector seem to suffer from the noise enhancement problems that often accompany zero-forcing detectors. Recall that the hybrid group-blind detector zero forces the interference of the known users and suppresses the interference from the unknown users via the MMSE criterion. Once again, note the relatively small difference between the performance of NAHJ-FST and exact SVD, especially at high SNR.

Fig. 9. Adaptive performance of NAHJ-FST.
F. Conclusion

In this chapter we have developed a new low-complexity, high-performance, subspace tracking algorithm and applied it to an asynchronous multipath version of the group-blind multiuser detection problem proposed in [40]. We have seen that at moderate and high SNR, the adaptive hybrid group-blind detector provides a substantial performance gain over the blind adaptive MMSE detector. The noise enhancement problems of the hybrid group-blind detector at low SNR suggest the use of a dual-mode receiver that uses the blind approach for low SNR environments and the group-blind approach for moderate and high SNR environments. We have also demonstrated that the performance of NAHJ-FST in the context of adaptive multiuser detection is similar to the performance of an exact rank-one SVD update, which serves as a performance
Fig. 11. Steady state performance of NAHJ-FST and exact SVD.

upper bound for all SVD-based subspace tracking algorithms. Table III summarizes the new algorithm.
Table III

**NAHJ-FST Subspace Tracking Algorithm.**

<table>
<thead>
<tr>
<th>Given: $\Sigma^2(l-1) = \begin{bmatrix} \Sigma^2_s(l-1) &amp; 0 \ 0 &amp; \sigma^2(l-1)I \end{bmatrix}, V_s(l-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Calculate $r_s, v_n,$ and $\beta$ according to (4.27) and (4.28).</td>
</tr>
<tr>
<td>2. Dropping the indices, generate the modified factorization</td>
</tr>
<tr>
<td>$[V_s</td>
</tr>
<tr>
<td>$\begin{bmatrix} [r_s^H</td>
</tr>
<tr>
<td>3. Let $R_s$ be the $r + 2$ principal submatrix of the matrix sum</td>
</tr>
<tr>
<td>in step 2. Apply a sequence of $r + 1$ Givens rotations to $R_s$ to</td>
</tr>
<tr>
<td>produce $R_a = \Theta_{r+1}^T \cdots \Theta_1^T R_s \Theta_1 \cdots \Theta_{r+1}.$</td>
</tr>
<tr>
<td>4. Let $A_s$ be the diagonal matrix whose diagonal is equal to</td>
</tr>
<tr>
<td>the first $r$ elements of the diagonal of $R_a.$</td>
</tr>
<tr>
<td>5. Let $U_s$ be composed of the first $r$ columns of</td>
</tr>
<tr>
<td>$[V_s</td>
</tr>
<tr>
<td>6. Set $\Sigma^2_s(l)$ equal to the $r + 1$ principal submatrix of $R_a.$</td>
</tr>
<tr>
<td>7. Let $V_s(l)$ be composed of the first $r + 1$ columns of</td>
</tr>
<tr>
<td>$[V_s</td>
</tr>
<tr>
<td>8. Reaverage the noise power: $\sigma^2(l) = \frac{(P_{m-r-2})(\sqrt{\gamma} \sigma^2(l-1)) +</td>
</tr>
<tr>
<td>where $\hat{\sigma}^2 = (R_a)_{r+2,r+2}.$</td>
</tr>
</tbody>
</table>
CHAPTER V

BLIND ADAPTIVE SPACE-TIME MULTIUSER DETECTION WITH MULTIPLE TRANSMITTER AND RECEIVER ANTENNAS

A. Introduction

One of the new technologies that is being considered for 3G and later generation wide-band standards is space-time processing. Generally speaking, space-time processing involves the exploitation of spatial diversity using multiple transmit and/or receive antennas and, perhaps, some form of coding. The initial focus was on systems that use one transmit antenna and multiple receive antennas [48, 49, 50, 51, 52, 53, 54]. Recently, however, much of the work in this area has focused on transmit diversity schemes that use multiple transmit antennas. They include delay schemes [4, 5, 6, 7] in which copies of the same symbol are transmitted through multiple antennas at different times, the space-time trellis coding algorithm developed by Tarokh, et. al. in [8], and the simple space-time block coding (STBC) scheme developed by Alamouti [9], which has been adopted in a number of 3G WCDMA standards [55, 56]. A generalization of the Alamouti space-time block coding concept is developed in [57]. It has been shown that these techniques can significantly improve capacity [58, 59].

Recently, some work has been completed on space-time multiuser detection using multiple antennas at both the transmitter and receiver. In [60], for example, the authors considered maximum-likelihood space-time multiuser detection in a CDMA system using orthogonal spreading codes. An application of space-time block coding to CDMA appears in [61]. However, this work assumes a perfectly-known channel and does not investigate blind adaptive algorithms or make use of the popular Alamouti space-time code. In the present work, we consider the performance of linear space-
time multiuser detection using multiple transmit and receive antennas for CDMA systems using non-orthogonal codes. First, we will compare two different linear receiver structures (linear diversity combining and space-time detection) for various antenna configurations. Motivated by the use of STBC in 3G proposals, we will utilize this block code for two-transmit-antenna configurations. Then we develop blind adaptive implementations of the two transmit/two receive antenna configuration for synchronous CDMA in flat fading channels and for asynchronous CDMA in fading multipath channels. The adaptive techniques developed here are blind, in the sense that the only information known to the receiver is the signature sequence of the user of interest.

The remainder of this chapter is organized as follows. In Section 2, we analyze and compare two different linear receiver structures that are appropriate for CDMA with multiple transmit and/or receive antennas. In Section 3, we develop blind adaptive implementations of the space-time receiver structure for synchronous CDMA in flat fading channels. In Section 4, we extend the sequential adaptive implementation to asynchronous CDMA in fading multipath channels. In Section 5 we present simulation results, and Section 6 concludes.

B. Space-Time Multiuser Detection in Synchronous CDMA: Analysis

In this section, we analyze receiver structures for synchronous CDMA systems with multiple transmitter antennas and multiple receiver antennas. Specifically, we focus on three configurations, namely, (1) one transmitter antenna, two receiver antennas; (2) two transmitter antennas, one receiver antenna; and (3) two transmitter antennas and two receiver antennas. It is assumed that a space-time block code is employed in systems with two transmitter antennas. For each of these configurations, we discuss
two possible linear receiver structures and compare their performance in terms of diversity gain and signal separation capability.

1. One Transmitter Antenna, Two Receiver Antennas

Consider the following discrete-time $K$-user synchronous CDMA channel with one transmitter antenna and two receiver antennas. The received baseband signal at the $p$-th antenna can be modeled as

$$ r_p = \sum_{k=1}^{K} h_{p,k} b_k s_k + n_p, \quad p = 1, 2. $$

(5.1)

where $s_k$ is the $N$-vector of the discrete-time signature waveform of the $k$-th user with unit norm, i.e., $\|s_k\| = 1$; $b_k \in \{+1, -1\}$ is the data bit of the $k$-th user; $h_{p,k}$ is the complex channel response of the $p$-th receiver antenna element to the $k$-th user’s signal; $n_p \sim \mathcal{N}(0, \sigma^2 I_N)$ is the ambient noise vector at antenna $p$. It is assumed that $n_1$ and $n_2$ are independent.

a. Linear Diversity Multiuser Detector

Denote

$$ h_k \triangleq [h_{1,k} \ h_{2,k}]^T $$

$$ S \triangleq [s_1 \ldots s_K] $$

$$ R \triangleq S^T S $$

Suppose that user 1 is the user of interest. We first consider the linear diversity multiuser detection scheme, which first applies a linear multiuser detector to the received signal $r_p$ in (5.1) at each antenna $p = 1, 2$, and then combines the outputs of these linear detectors to make a decision. For example, a linear decorrelating detector
for user 1 based on the signal in (5.1) is simply [62]

$$w_1 = SR^{-1}e_1, \quad (5.2)$$

where $e_1$ denotes the first unit vector in $\mathbb{R}^K$. This detector is applied to the received signal at each antenna $p = 1, 2$, to obtain $z = [z_1 \ z_2]^T$, where

$$z_p \triangleq w_1^T r_p = h_{p,1} b_1 + u_p, \quad (5.3)$$

with

$$u_p \triangleq w_1^T n_p \sim \mathcal{N}_c \left(0, \sigma^2 \|w_1\|^2\right), \quad p = 1, 2 \quad (5.4)$$

where $\|w_1\|^2 = [R^{-1}]_{1,1}$ and where $[A]_{i,j}$ denotes the element in the $i$-th row and $j$-th column of the matrix $A$. Denote

$$\eta_1 \triangleq \frac{1}{\sqrt{[R^{-1}]_{1,1}}}. \quad (5.5)$$

Denote $h_k \triangleq [h_{1,k} \ h_{2,k}]^T$. Since the noise vectors from different antennas are independent, we can write

$$z = b_1 h_1 + u, \quad (5.6)$$

with

$$u \sim \mathcal{N}_c \left(0, \frac{\sigma^2}{\eta_1^2} \cdot I_2 \right). \quad (5.7)$$

The maximum likelihood (ML) decision rule for $b_1$ based on $z$ in (5.6) is then

$$\hat{b}_1 = \operatorname{sign} \left(\Re \{h_1^H z\} \right). \quad (5.8)$$

Let $E_1 \triangleq h_1^H h_1$ be the total received desired user’s signal energy. The decision statistic in (5.8) can be expressed as

$$\xi \triangleq h_1^H z = E_1 b_1 + v, \quad (5.9)$$

with

$$v \triangleq h_1^H u \sim \mathcal{N}_c \left(0, E_1 \sigma^2 / \eta_1^2 \right). \quad (5.10)$$
The probability of detection error of the linear diversity multiuser detector is computed as

\[
P_D^1(e) = P\left(\Re\{\xi\} < 0 \mid b_1 = 1\right) = P\left(\Re\{v\} < -E_1\right) = Q\left(\frac{\sqrt{2E_1}}{\sigma} \cdot \eta_1\right).
\] (5.11)

b. Linear Space-Time Multiuser Detector

Denote

\[
\begin{align*}
b & \triangleq [b_1 \cdots b_K]^T \\
H & \triangleq [h_1 \cdots h_K] \\
\tilde{s}_k & \triangleq h_k \otimes s_k \\
\tilde{S} & \triangleq [\tilde{s}_1 \cdots \tilde{s}_K] \\
\tilde{R} & \triangleq \tilde{S}^H \tilde{S} \\
\tilde{r} & \triangleq [r_1^T r_2^T]^T \\
\tilde{n} & \triangleq [n_1^T n_2^T]^T,
\end{align*}
\]

where \( \otimes \) denotes the Kronecker product. Then by augmenting the received signals at two antennas, (5.1) can be written as

\[
\tilde{r} = \sum_{k=1}^{K} b_k \tilde{s}_k + \tilde{n} = \tilde{S}b + \tilde{n},
\] (5.12)

with \( \tilde{n} \sim \mathcal{N}_c(0, \sigma^2 I_{2N}) \). A linear space-time multiuser detector operates on the augmented received signal \( \tilde{r} \) directly. For example, the linear decorrelating detector for user 1 in this case is given by

\[
\tilde{w}_1 = \tilde{S} \tilde{R}^{-1} e_1.
\] (5.13)
This detector is applied to the augmented received signal $\tilde{r}$ to obtain

$$\tilde{z} \triangleq \tilde{w}^H \tilde{r} = b_1 + \tilde{u},$$

with $\tilde{u} \triangleq \tilde{w}^H \tilde{n} \sim \mathcal{N}_c(0, \sigma^2\|\bar{w}_1\|^2)$,

where $\|\bar{w}_1\|^2 = \left[ \tilde{R}^{-1} \right]_{1,1}$. Denote

$$\tilde{\eta}_1 \triangleq \frac{1}{\sqrt{E_1}} \cdot \frac{1}{\sqrt{\left[ \tilde{R}^{-1} \right]_{1,1}}}.$$ \hspace{1cm} (5.16)

An expression for $\tilde{R}$ can be found as follows. Note that

$$[\tilde{R}]_{i,j} = \left[ S^H \tilde{S} \right]_{i,j} = \tilde{s}_i^H \tilde{s}_j = (h_i \otimes s_i)^H (h_j \otimes s_j) = (h_i^H \otimes s_i^T) (h_j \otimes s_j) = (h_i^H h_j) \otimes (s_i^T s_j) = [H^H H]_{i,j} \cdot [R]_{i,j}.$$ \hspace{1cm} (5.17)

Hence

$$\tilde{R} \triangleq \tilde{S}^H \tilde{S} = R \circ (H^H H),$$

where $\circ$ denotes the Schur matrix product (i.e., element-wise product).

The ML decision rule for $b_1$ based on $\tilde{z}$ in (5.14) is then

$$\tilde{b}_1 = \text{sign}(\Re \{\tilde{z}\}).$$

(5.19)

The probability of detection error of the space time multiuser detector is computed as

$$P_{ST}^1(e) = P\left(\Re \{\tilde{z}\} < 0 \mid b_1 = 1\right)$$

$$= P\left(\Re \{\tilde{u}\} < -1\right) = Q\left(\frac{\sqrt{2E_1}}{\sigma} \cdot \tilde{\eta}_1\right).$$ \hspace{1cm} (5.20)
c. Performance Comparison

From the above discussion it is seen that the linear space-time multiuser detector exploits the signal structure in both the time domain (i.e., induced by the signature waveform $s_k$) and the spatial domain (i.e., induced by the channel response $h_k$) for interference rejection; whereas for the linear diversity multiuser detector, interference rejection is performed only in the time domain, and the spatial domain is only used for diversity combining. The next result, first appearing in [53], shows that the linear space-time multiuser detector always outperforms the linear diversity multiuser detector. The proof appears in the Appendix.

**Proposition 1.** Let $P^D_1(e)$ given by (5.11) and $P^{ST}_k(e)$ given by (5.20) be respectively the probability of detection error of the linear diversity detector and the linear space-time detector. Then

$$P^{ST}_1(e) \leq P^D_1(e).$$

We next consider a simple example to demonstrate the performance difference between the two receivers discussed above. Consider a 2-user system with

$$R = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 1 \\ e^{j\theta_1} & e^{j\theta_2} \end{bmatrix},$$

where $\rho$ is the correlation of the signature waveforms of the two users’; and $\theta_1$ and $\theta_2$ are the directions of arrival of the two users’ signals. Denote $\alpha \triangleq \theta_2 - \theta_1$. Then we
have $E_1 = E_2 = 1$, and

$$Q \triangleq \Phi H \Phi = \begin{bmatrix} 2 & 1 + e^{j\alpha} \\ 1 + e^{-j\alpha} & 2 \end{bmatrix}, \quad (5.21)$$

$$\bar{R} \triangleq R \circ Q = \begin{bmatrix} 2 & \rho (1 + e^{j\alpha}) \\ \rho (1 + e^{-j\alpha}) & 2 \end{bmatrix}, \quad (5.22)$$

$$\eta_1 \triangleq \frac{1}{\sqrt{[R^{-1}]_{1,1}}} = \sqrt{1 - \rho^2}, \quad (5.23)$$

$$\bar{\eta}_1 \triangleq \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{[R^{-1}]_{1,1}}} = \sqrt{1 - \rho^2 \cos^2 \frac{\alpha}{2}}, \quad (5.24)$$

It is seen that while the multiuser space-time receiver can exploit both the temporal signal separation (along $\rho$-axis) and the spatial signal separation (along $\alpha$-axis), the multiuser diversity receiver can only exploit the temporal signal separation. For example, for large $\rho$, the performance of the multiuser diversity receiver is poor, no matter what value $\alpha$ takes; but the performance of the multiuser space-time receiver can be quite good as long as $\alpha$ is large.

2. Two Transmitter Antennas, One Receiver Antenna

When two antennas are employed at the transmitter, we must first specify how the information bits are transmitted across the two antennas. Here we adopt the Alamouti space-time block coding scheme [9, 57]. Specifically, for each user $k$, two information symbols $b_{1,k}$ and $b_{2,k}$ are transmitted over two symbol intervals. At the first time interval, the symbol pair $(b_{1,k}, b_{2,k})$ are transmitted across the two transmitter antennas; and at the second time interval, the symbol pair $(-b_{2,k}, b_{1,k})$ are transmitted.
The received signal corresponding to these two time intervals are given by

\begin{align*}
r_1 &= \sum_{k=1}^{K} \left( h_{1,k} b_{1,k} + h_{2,k} b_{2,k} \right) s_k + n_1, \\
r_2 &= \sum_{k=1}^{K} \left( -h_{1,k} b_{2,k} + h_{2,k} b_{1,k} \right) s_k + n_2,
\end{align*}

(5.25) (5.26)

where \( h_{1,k} \) (\( h_{2,k} \)) is the complex channel responses between the first (second) transmitter antenna and the receiver antenna; \( n_1 \) and \( n_2 \) are independent received \( \mathcal{CN}(0, I_N) \) noise vectors at the two time intervals.

\textbf{a. Linear Diversity Multiuser Detector}

We first consider the linear diversity multiuser detection scheme, which first applies the linear multiuser detector \( \mathbf{w}_1 \) in (5.2) to the received signals \( r_1 \) and \( r_2 \) during the two time intervals, and then performs a space-time decoding. Specifically, denote

\begin{align*}
\mathbf{z}_1 &\triangleq \mathbf{w}_1^T \mathbf{r}_1 = h_{1,k} b_{1,k} + h_{2,k} b_{2,k} + u_1, \\
\mathbf{z}_2 &\triangleq (\mathbf{w}_1^T \mathbf{r}_2)^* = -h_{1,k}^* b_{2,k} + h_{2,k}^* b_{1,k} + u_2^*, \\
\text{with} \quad u_p &\triangleq \mathbf{w}_1^T \mathbf{n}_p \sim \mathcal{CN}(0, \sigma^2 \| \mathbf{w}_1 \|^2), \quad p = 1, 2.
\end{align*}

(5.27) (5.28) (5.29)

where \( \| \mathbf{w}_1 \|^2 = [R^{-1}]_{1,1} \).

Denote

\begin{align*}
\mathbf{z} &\triangleq \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}, \quad \mathbf{u} \triangleq \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2^* \end{bmatrix}, \quad \mathbf{h}_k \triangleq \begin{bmatrix} h_{1,k} \\ h_{2,k}^* \end{bmatrix}, \quad \hat{\mathbf{h}}_k \triangleq \begin{bmatrix} h_{2,k} \\ -h_{1,k}^* \end{bmatrix}.
\end{align*}
It is easily seen that $h_k^H \bar{h}_k = 0$. Then (5.27)-(5.29) can be written as

$$z = [h_1 \bar{h}_1] \begin{bmatrix} b_{1,1} \\ b_{2,1} \end{bmatrix} + u, \quad (5.30)$$

with $u \sim \mathcal{N}_c \left(0, \frac{\sigma^2}{\eta_1^2} \cdot I_2\right). \quad (5.31)$

As before, denote $E_1 \triangleq h_1^H \bar{h}_1 = \bar{h}_1^H h_1$. Note that

$$[h_1 \bar{h}_1]^H [h_1 \bar{h}_1] = \begin{bmatrix} E_1 & 0 \\ 0 & E_1 \end{bmatrix}. \quad (5.32)$$

The ML decision rule for $b_{1,1}$ and $b_{2,1}$ based on $z$ in (5.30) is then given by

$$\begin{bmatrix} \hat{b}_{1,1} \\ \hat{b}_{2,1} \end{bmatrix} = \text{sign} \left( \Re \left\{ [h_1 \bar{h}_1]^H z \right\} \right) = \text{sign} \left( \Re \left\{ \left[ h_1^H z \right] \right\} \right). \quad (5.33)$$

Using (5.30), it is easily seen that the decision statistic in (5.33) is distributed according to

$$\frac{1}{\sqrt{E_1}} h_1^H z \sim \mathcal{N}_c \left( \sqrt{E_1} b_{1,1}, \frac{\sigma^2}{\eta_1^2} \right). \quad (5.34)$$

$$\frac{1}{\sqrt{E_1}} \bar{h}_1^H z \sim \mathcal{N}_c \left( \sqrt{E_1} b_{2,1}, \frac{\sigma^2}{\eta_1^2} \right). \quad (5.35)$$

Hence the probability of error is given by

$$P_1^D(e) = Q \left( \frac{\sqrt{2E_1}}{\sigma} \cdot \eta_1 \right). \quad (5.36)$$

This is the same expression as (5.20) for the linear diversity receiver with one transmitter antenna and two receiver antennas.
b. Linear Space-Time Multiuser Detector

Denote

\[ \tilde{r} = \begin{bmatrix} r_1 \\ r_2^* \end{bmatrix}, \quad \tilde{n} = \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}. \]

Then (5.25) and (5.26) can be written as

\[ \tilde{r} = \sum_{k=1}^{K} \left( b_{1,k} h_k \otimes s_k + b_{2,k} \bar{h}_k \otimes s_k \right) + \tilde{n}. \]  

(5.37)

Denote

\[ \tilde{S} = \left[ h_1 \otimes s_1, \ h_1 \otimes s_1, \ldots, \ h_K \otimes s_K, \ \bar{h}_K \otimes s_K \right]_{2N \times 2K}, \]

(5.38)

\[ \tilde{R} = \tilde{S}^H \tilde{S}. \]  

(5.39)

Then the decorrelating detector for detecting the bit \( b_{1,1} \) based on \( \tilde{r} \) in (5.37) is given by

\[ \tilde{w}_{1,1} = \tilde{S} \tilde{R}^{-1} \tilde{e}_1, \]  

(5.40)

where \( \tilde{e}_1 \) is the first unit vector in \( \mathbb{R}^{2K} \).

**Proposition 2.** The decorrelating detector in (5.40) is given by

\[ \tilde{w}_{1,1} = \frac{h_1 \otimes w_1}{\|h_1\|^2}, \]  

(5.41)

where \( w_1 \) is given by (5.2).

**Proof:** We need to verify that

\[ \left( \frac{h_1 \otimes w_1}{\|h_1\|^2} \right)^H \tilde{S} = \tilde{e}_1. \]  

(5.42)
We have

\[
\frac{1}{\|h_1\|^2} (h_1 \otimes w_1)^H (h_1 \otimes s_1) = \frac{1}{\|h_1\|^2} (h_1^H h_1) \begin{pmatrix} w_1^T s_1 \end{pmatrix} = 1
\] (5.43)

\[
\frac{1}{\|h_1\|^2} (h_1 \otimes w_1)^H (\tilde{h}_1 \otimes s_1) = \frac{1}{\|h_1\|^2} \frac{1}{\|h_1\|^2} \begin{pmatrix} h_1^H \tilde{h}_1 \end{pmatrix} \begin{pmatrix} w_1^T s_1 \end{pmatrix} = 0
\] (5.44)

\[
\frac{1}{\|h_1\|^2} (h_1 \otimes w_1)^H (h_k \otimes s_k) = \frac{1}{\|h_1\|^2} \begin{pmatrix} h_1^H h_k \end{pmatrix} \begin{pmatrix} w_1^T s_k \end{pmatrix} = 0, k = 2, \cdots, K
\] (5.45)

\[
\frac{1}{\|h_1\|^2} (h_1 \otimes w_1)^H (\tilde{h}_k \otimes s_k) = \frac{1}{\|h_1\|^2} \begin{pmatrix} h_1^H \tilde{h}_k \end{pmatrix} \begin{pmatrix} w_1^T s_k \end{pmatrix} = 0, k = 2, \cdots, K
\] (5.46)

This verifies (5.42) so that (5.41) is indeed the decorrelating detector given by (5.40)

\[\square\]

Hence the output of the linear space-time detector in this case is given by

\[
\tilde{z}_1 = \tilde{w}_{1,1}^H \tilde{r} = b_{1,1} + u_1
\] (5.47)

with

\[
u_1 \triangleq \tilde{w}_{1,1}^H \tilde{n} \sim \mathcal{N}_c(0, \sigma^2\|\tilde{w}_{1,1}\|^2)
\] (5.48)

where using (5.5) and (5.41), we have

\[
\|\tilde{w}_{1,1}\|^2 = \frac{\|h_1 \otimes w_1\|^2}{\|h_1\|^4} = \frac{\|w_1\|^2}{\|h_1\|^2} = \frac{1}{E_1 \eta_1^2}.
\] (5.49)

Therefore the probability of detection error is given by

\[
P_{ST}^i(e) = P\left( \Re\{\tilde{z}_1\} < 0 \mid b_{1,1} = 1 \right) = P\left( \Re\{u_1\} < -1 \right) = Q\left( \frac{\sqrt{2E_1}}{\sigma} \cdot \eta_1 \right).
\] (5.50)

Compare (5.36) with (5.50) we see that for the case of two transmitter antennas and one receiver antenna, the linear diversity receiver and the linear space-time receiver have the same performance. Hence the multiple transmitter antennas with space-time block coding only provide diversity gain, but no signal separation capability.
3. Two Transmitter and Two Receiver Antennas

We combine the results from the previous two sections to investigate an environment in which we use two transmitter antennas and two receiver antennas. We adopt the space-time block coding scheme used in the previous section. The received signal at antenna 1 during the two symbol intervals is

\[
\begin{align*}
\mathbf{r}^{(1)}_1 &= \sum_{k=1}^{K} \left[ h^{(1,1)}_{k} b_{1,k} + h^{(2,1)}_{k} b_{2,k} \right] \mathbf{s}_k + \mathbf{n}^{(1)}_1, \\
\mathbf{r}^{(1)}_2 &= \sum_{k=1}^{K} \left[ -h^{(1,1)}_{k} b_{2,k} + h^{(2,1)}_{k} b_{1,k} \right] \mathbf{s}_k + \mathbf{n}^{(1)}_2,
\end{align*}
\]

(5.51)

(5.52)

and the corresponding signals received at antenna 2 are

\[
\begin{align*}
\mathbf{r}^{(2)}_1 &= \sum_{k=1}^{K} \left[ h^{(1,2)}_{k} b_{1,k} + h^{(2,2)}_{k} b_{2,k} \right] \mathbf{s}_k + \mathbf{n}^{(2)}_1, \\
\mathbf{r}^{(2)}_2 &= \sum_{k=1}^{K} \left[ -h^{(1,2)}_{k} b_{2,k} + h^{(2,2)}_{k} b_{1,k} \right] \mathbf{s}_k + \mathbf{n}^{(2)}_2,
\end{align*}
\]

(5.53)

(5.54)

where \( h^{(i,j)}_k, i, j \in \{1, 2\} \) is the complex channel response between transmitter antenna \( i \) and receiver antenna \( j \) for user \( k \). The noise vectors \( \mathbf{n}^{(1)}_1, \mathbf{n}^{(2)}_1, \mathbf{n}^{(1)}_2, \) and \( \mathbf{n}^{(2)}_2 \) are independent and identically distributed with distribution \( \mathcal{N}_c(0, \sigma^2 I_N) \).

a. Linear Diversity Multiuser Detector

As before, we first consider the linear diversity multiuser detection scheme for user 1, which applies the linear multiuser detector \( \mathbf{w}_1 \) in (5.2) to each of the four received signals \( \mathbf{r}^{(1)}_1, \mathbf{r}^{(2)}_1, \mathbf{r}^{(1)}_2, \) and \( \mathbf{r}^{(2)}_2 \) and then performs a space-time decoding. Specifically,
denote the filter outputs as

\[ z^{(1)}_1 \triangleq w_1^T r^{(1)}_1 = h^{(1,1)}_1 b_{1,1} + h^{(2,1)}_1 b_{2,1} + u^{(1)}_1, \quad (5.55) \]

\[ z^{(1)}_2 \triangleq \left( w_1^T r^{(1)}_2 \right)^* = -\left( h^{(1,1)}_1 \right)^* b_{2,1} + \left( h^{(2,1)}_1 \right)^* b_{1,1} + \left( u^{(1)}_2 \right)^*, \quad (5.56) \]

\[ z^{(2)}_1 \triangleq w_1^T r^{(2)}_1 = h^{(1,2)}_1 b_{1,1} + h^{(2,2)}_1 b_{2,1} + u^{(2)}_1, \quad (5.57) \]

\[ z^{(2)}_2 \triangleq \left( w_1^T r^{(2)}_2 \right)^* = -\left( h^{(1,2)}_1 \right)^* b_{2,1} + \left( h^{(2,2)}_1 \right)^* b_{1,1} + \left( u^{(2)}_2 \right)^*, \quad (5.58) \]

with \[ u^{(j)}_i \triangleq w_1^T n^{(j)}_i \sim N_c \left( 0, \frac{\sigma^2}{\eta_i^2} \right), \quad i,j = 1,2 \quad (5.59) \]

where, as before, \( \eta_i^2 \triangleq 1/ \left[ R^{-1} \right]_{1,1} \).

We define the following quantities:

\[ z \triangleq \begin{bmatrix} z^{(1)}_1 & z^{(1)}_2 & z^{(2)}_1 & z^{(2)}_2 \end{bmatrix}^T \]

\[ u \triangleq \begin{bmatrix} u^{(1)}_1 & (u^{(1)}_2)^* & u^{(2)}_1 & (u^{(2)}_2)^* \end{bmatrix}^T \]

\[ h^{(1)}_1 \triangleq \begin{bmatrix} h^{(1,1)}_1 & h^{(2,1)}_1 \end{bmatrix}^H \]

\[ \tilde{h}^{(1)}_1 \triangleq \begin{bmatrix} h^{(2,1)}_1 - h^{(1,1)}_1 \end{bmatrix}^T \]

\[ h^{(2)}_1 \triangleq \begin{bmatrix} h^{(1,2)}_1 & h^{(2,2)}_1 \end{bmatrix}^H \]

\[ \tilde{h}^{(2)}_1 \triangleq \begin{bmatrix} h^{(2,2)}_1 - h^{(1,2)}_1 \end{bmatrix}^T. \]

Then (5.55) - (5.59) can be written as

\[ z = \underbrace{\begin{bmatrix} h^{(1)}_1 & \tilde{h}^{(1)}_1 & h^{(2)}_1 & \tilde{h}^{(2)}_1 \end{bmatrix}^H}_{\tilde{H}^H_1} \begin{bmatrix} b_{1,1} \\ b_{2,1} \end{bmatrix} + u, \quad (5.60) \]

with \( u \sim N_c \left( 0, \frac{\sigma^2}{\eta_i^2} I_4 \right). \quad (5.61) \)
It is readily verified that
\[
H_1 H_1^H = \begin{bmatrix} E_1 & 0 \\ 0 & E_1 \end{bmatrix},
\]
with
\[
E_1 \triangleq |h_1^{(1,1)}|^2 + |h_1^{(1,2)}|^2 + |h_1^{(2,1)}|^2 + |h_1^{(2,2)}|^2.
\]
(5.62)

To form the ML decision statistic, we premultiply \( z \) by \( H_1 \) and obtain
\[
\begin{bmatrix} d_{1,1} \\ d_{2,1} \end{bmatrix} \triangleq H_1 z = E_1 \begin{bmatrix} b_{1,1} \\ b_{2,1} \end{bmatrix} + \nu,
\]
with \( \nu \sim \mathcal{N}(0, E_1 \sigma^2 \eta_1^2 \cdot I_2) \).
(5.64)

The corresponding bit estimates are given by
\[
\begin{bmatrix} \hat{b}_{1,1} \\ \hat{b}_{2,1} \end{bmatrix} = \text{sign} \left( \Re \left\{ \begin{bmatrix} d_{1,1} \\ d_{2,1} \end{bmatrix} \right\} \right).
\]
(5.66)

The bit error probability is then given by
\[
P_1^D(e) = P \left( \Re \{d_{1,1}\} < 0 \mid b_{1,1} = +1 \right)
= P \left[ E_1 + \mathcal{N} \left( 0, \frac{E_1 \sigma^2}{2\eta_1^2} \right) < 0 \right] = Q \left( \frac{\sqrt{2E_1}}{\sigma} \cdot \eta_1 \right).
\]
(5.67)

b. Linear Space-Time Multiuser Detector

We denote
\[
\tilde{r} \triangleq \begin{bmatrix} r_1^{(1)} \\ \ast(r_2^{(1)}) \\ r_1^{(2)} \\ \ast(r_2^{(2)}) \end{bmatrix}, \quad \tilde{n} \triangleq \begin{bmatrix} n_1^{(1)} \\ \ast(n_2^{(1)}) \\ n_1^{(2)} \\ \ast(n_2^{(2)}) \end{bmatrix}, \quad h_k \triangleq \begin{bmatrix} h_k^{(1,1)} \\ \ast(h_k^{(1,2)}) \\ h_k^{(2,1)} \\ \ast(h_k^{(2,2)}) \end{bmatrix}, \quad \tilde{h}_k \triangleq \begin{bmatrix} h_k^{(1,1)} \\ \ast(h_k^{(1,2)}) \\ h_k^{(2,1)} \\ \ast(h_k^{(2,2)}) \end{bmatrix}.
\]
Then (5.51)-(5.54) may be written as

\[
\tilde{r} = \sum_{k=1}^{K} \left( b_{1,k} h_k \otimes s_k + b_{2,k} \bar{h}_k \otimes s_k \right) + \tilde{n} = \tilde{S} b + \tilde{n},
\]

(5.68)

where

\[
\tilde{S} \triangleq \begin{bmatrix} h_1 \otimes s_1, \bar{h}_1 \otimes s_1, \ldots, h_K \otimes s_K, \bar{h}_K \otimes s_K \end{bmatrix}_{4N \times 2K}
\]

(5.69)

\[
b \triangleq \begin{bmatrix} b_{1,1} & b_{2,1} & b_{1,2} & \cdots & b_{1,K} & b_{2,K} \end{bmatrix}^T.
\]

(5.70)

Since \( h_k^H \bar{h}_k = 0 \) and (5.68) has the same form as (5.37), it is easy to show that the decorrelating detector for detecting the bit \( b_{1,1} \) based on \( \tilde{r} \) is given by

\[
\tilde{w}_{1,1} = \frac{h_1 \otimes w_1}{\|h_1\|^2}.
\]

(5.71)

Hence the output of the linear space-time detector in this case is given by

\[
\tilde{z}_1 = \tilde{w}_{1,1}^H \tilde{r} = b_{1,1} + u_1
\]

(5.72)

with

\[
u_1 \triangleq \tilde{w}_{1,1}^H \tilde{n} \sim \mathcal{N}(0, \sigma^2 \|\tilde{w}_{1,1}\|^2)
\]

(5.73)

where

\[
\|\tilde{w}_{1,1}\|^2 = \frac{\|w_1\|^2}{\|h_1\|^2} = \frac{1}{E_1 \eta_1^2}.
\]

(5.74)

Therefore the probability of detector error is given by

\[
P_{1}^{ST}(e) = P\left( \Re \{\tilde{z}_1\} < 0 \mid b_{1,1} = +1 \right)
\]

\[
= P \left[ 1 + \mathcal{N} \left( 0, \frac{1}{2E_1 \eta_1^2} \right) < 0 \right] = Q\left( \frac{\sqrt{2E_1}}{\sigma} \cdot \eta_1 \right).
\]

(5.75)

Comparing (5.75) with (5.67) it is seen that when two transmitter antennas and two receiver antennas are employed and the signals are transmitted in the form of space-time block code, then the linear diversity receiver and the linear space-time receiver
have identical performance.

4. Remarks

We have seen that the performance of space-time multiuser detection (STMUD) and linear diversity multiuser detection (LDMUD) are similar for two transmit/one receive and two transmit/two receive antenna configurations. What, then, are the benefits of the space-time detection technique? They are as follows:

1. Although LDMUD and STMUD perform similarly for the 2-1 and 2-2 cases, the performance of STMUD is superior for configurations with one transmit antenna and \( P \geq 2 \) receive antennas.

2. User capacity for CDMA systems is limited by correlations among composite signature waveforms. This multiple-access interference will tend to decrease as the dimension of the vector space in which the signature waveforms reside increases. The signature waveforms for linear diversity detection are of length \( N \), i.e., they reside in \( \mathbb{C}^N \). Since the received signals are stacked for space-time detection, these signature waveforms reside in \( \mathbb{C}^{2N} \) for two transmit and one receive antenna or \( \mathbb{C}^{4N} \) for two transmit and two receive antennas. As a result, the space-time structure can support more users than linear diversity detection for a given performance threshold.

3. For adaptive configurations (Section b and Section 4), LDMUD requires four independent subspace trackers operating simultaneously since the receiver performs detection on each of the four received signals, and each has a different signal subspace. The space-time structure requires only one subspace tracker.
C. Blind Adaptive Implementations of Space-Time Multiuser Detection for Synchronous CDMA

We next develop both batch and sequential *blind adaptive* implementations of the linear space-time receiver. These implementations are blind in the sense that they require only knowledge of the signature waveform of the user of interest. Instead of the decorrelating detector used for analysis in the previous section, we will use a linear MMSE detector for the adaptive implementations because the MMSE detector is more suitable for adaptation and its performance is comparable to that of the decorrelating detector. This is reasonable since the detectors are asymptotically identical as the AWGN power tends to zero and they share the same near-far resistance. We consider only the environment in which we have two transmitter antennas and two receiver antennas. The other cases can be derived in a similar manner. Note that inherent to any *blind* receiver in multiple transmitter antenna systems is an ambiguity issue. That is, if the same spreading waveform is used for a user at both transmitter antennas, the blind receiver can not distinguish which bit is from which antenna. To resolve such an ambiguity, here we use two different spreading waveforms for each user, i.e. \( s_{j,k}, j \in \{1, 2\} \) is the spreading code for user \( k \) for the transmission of bit \( b_{j,k} \).

There are two bits, \( b_{1,k}[i] \) and \( b_{2,k}[i] \), associated with each user at each time slot \( i \) and the difference in time between slots is \( 2T_s \) where \( T_s \) is the symbol period. The received signal at antenna 1 during the two symbol periods for time slot \( i \) is

\[
\begin{align*}
\mathbf{r}_1^{(1)}[i] &= \sum_{k=1}^{K} \left( h_k^{(1,1)} b_{1,k}[i] \mathbf{s}_{1,k} + h_k^{(2,1)} b_{2,k}[i] \mathbf{s}_{2,k} \right) + \mathbf{n}_1^{(1)}[i], \\
\mathbf{r}_2^{(1)}[i] &= \sum_{k=1}^{K} \left( -h_k^{(1,1)} b_{2,k}[i] \mathbf{s}_{2,k} + h_k^{(2,1)} b_{1,k}[i] \mathbf{s}_{1,k} \right) + \mathbf{n}_2^{(1)}[i],
\end{align*}
\]
and the corresponding signals received at antenna 2 are

\[
\begin{align*}
\mathbf{r}_1^{(2)}[i] &= \sum_{k=1}^{K} \left( h_k^{(1,2)} b_{1,k}[i] s_{1,k} + h_k^{(2,2)} b_{2,k}[i] s_{2,k} \right) + \mathbf{n}_1^{(2)}[i], \\
\mathbf{r}_2^{(2)}[i] &= \sum_{k=1}^{K} \left( -h_k^{(1,2)} b_{2,k}[i] s_{2,k} + h_k^{(2,2)} b_{1,k}[i] s_{1,k} \right) + \mathbf{n}_2^{(2)}[i].
\end{align*}
\]

We stack these received signal vectors and denote

\[
\begin{align*}
\mathbf{\bar{r}}[i] &\triangleq \begin{bmatrix} \mathbf{r}_1^{(1)}[i] \\ \mathbf{r}_1^{(2)}[i]^* \\ \mathbf{r}_2^{(1)}[i] \\ \mathbf{r}_2^{(2)}[i]^* \end{bmatrix}, & \mathbf{\bar{n}}[i] &\triangleq \begin{bmatrix} \mathbf{n}_1^{(1)}[i] \\ \mathbf{n}_2^{(1)}[i]^* \\ \mathbf{n}_1^{(2)}[i] \\ \mathbf{n}_2^{(2)}[i]^* \end{bmatrix}, \\
\mathbf{h}_k &\triangleq \begin{bmatrix} h_k^{(1,1)} \\ (h_k^{(1,2)})^* \\ h_k^{(2,1)} \\ (h_k^{(2,2)})^* \end{bmatrix}, & \mathbf{\bar{h}}_k &\triangleq \begin{bmatrix} h_k^{(2,1)} \\ (-h_k^{(1,1)})^* \\ h_k^{(2,2)} \\ (-h_k^{(1,2)})^* \end{bmatrix},
\end{align*}
\]

Then we may write

\[
\mathbf{\bar{r}}[i] = \sum_{k=1}^{K} \left( b_{1,k}[i] \mathbf{h}_k \otimes \mathbf{s}_{1,k} + b_{2,k}[i] \mathbf{\bar{h}}_k \otimes \mathbf{s}_{2,k} \right) + \mathbf{\bar{n}}[i] = \mathbf{\bar{S}} \mathbf{b}[i] + \mathbf{\bar{n}}[i],
\]

where

\[
\begin{align*}
\mathbf{\bar{S}} &\triangleq \left[ \mathbf{h}_1 \otimes \mathbf{s}_{1,1}, \ \mathbf{h}_1 \otimes \mathbf{s}_{2,1}, \ldots \mathbf{h}_K \otimes \mathbf{s}_{1,K}, \ \mathbf{\bar{h}}_K \otimes \mathbf{s}_{2,K} \right]_{4N \times 2K} \\
\mathbf{b}[i] &\triangleq \left[ b_{1,1}[i] \ b_{2,1}[i] \ b_{1,2}[i] \ b_{2,2}[i] \cdots b_{1,K}[i] \ b_{2,K}[i] \right]^T_{2K \times 1}.
\end{align*}
\]

The autocorrelation matrix of the stacked signal \(\mathbf{\bar{r}}[i]\), \(\mathbf{C}\), and its eigendecomposition
are given by

$$
\mathbf{C} = E \{ \mathbf{r}[i] \mathbf{r}[i]^H \} = \mathbf{S}\mathbf{S}^H + \sigma^2 \mathbf{I}_{4N}
$$

(5.81)

$$
= \mathbf{U}_s \mathbf{A}_s \mathbf{U}_s^H + \sigma^2 \mathbf{U}_n \mathbf{U}_n^H,
$$

(5.82)

where \( \mathbf{A}_s = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_{2K}\} \) contains the largest \((2K)\) eigenvalues of \( \mathbf{C} \), the columns of \( \mathbf{U}_s \) are the corresponding eigenvectors; and the columns of \( \mathbf{U}_n \) are the \((4N - 2K)\) eigenvectors corresponding to the smallest eigenvalue \( \sigma^2 \).

The blind linear MMSE detector for detecting \( [\mathbf{b}[i]]_1 = b_{1,1}[i] \) is given by the solution to the optimization problem

$$
\mathbf{w}_{1,1} \triangleq \arg \min_{\mathbf{w} \in \mathbb{C}^{4N}} E \left\{ |b_{1,1}[i] - \mathbf{w}^H \tilde{\mathbf{r}}[i]|^2 \right\}.
$$

(5.83)

It has been shown in [10] that a scaled version of the solution can be written in terms of the signal subspace components as

$$
\mathbf{w}_{1,1} = \mathbf{U}_s \mathbf{A}_s^{-1} \mathbf{U}_s^H (\mathbf{h}_1 \otimes \mathbf{s}_{1,1}),
$$

(5.84)

and the decision is made according to

$$
\hat{z}_{1,1}[i] = \mathbf{w}_{1,1}^H \tilde{\mathbf{r}}[i],
$$

(5.85)

$$
\hat{b}_{1,1}[i] = \text{sign} \left[ \Re \left( \hat{z}_{1,1}[i] \right) \right] \quad \text{(coherent detection)}
$$

(5.86)

$$
\hat{\beta}_{1,1}[i] = \text{sign} \left[ \Re \left( z_{1,1}[i-1]^* z_{1,1}[i] \right) \right] \quad \text{(differential detection)}.
$$

(5.87)

Before we address specific batch and sequential adaptive algorithms, we note that these algorithms can be also be implemented using linear \emph{group-blind} multiuser detectors instead of blind MMSE detectors. This would be appropriate, for example, in uplink environments in which the base station has knowledge of the signature waveforms of all of the users in the cell, but not those of users outside the cell.
Specifically, we may rewrite (5.80) as

\[ \mathbf{r}[i] = \mathbf{\bar{S}} \mathbf{b}[i] + \mathbf{\bar{S}} \mathbf{b}[i] + \mathbf{n}[i], \]  

(5.88)

where we have separated the users into two groups. The composite signature sequences of the known users are the columns of \( \mathbf{\bar{S}} \). The unknown users’ composite sequences are the columns of \( \mathbf{\bar{S}} \). Then the group-blind linear hybrid detector for bit \( b_{1,1}[i] \) is given by [3]

\[ \mathbf{w}_{1,1}^{GB} = \mathbf{U_s A_s^{-1} U_s^H \bar{S}} \left[ \mathbf{\bar{S}}^H \mathbf{U_s A_s^{-1} U_s^H \bar{S}} \right]^{-1} (\mathbf{h}_1 \otimes \mathbf{s}_{1,1}). \]  

(5.89)

This detector offers a significant performance improvement over blind implementations of (5.84) for environments in which the signature sequences of some of the interfering users are known.

a. Batch Blind Linear Space-Time Multiuser Detection

In order to obtain an estimate of \( \mathbf{h}_1 \) we make use of the orthogonality between the signal and noise subspaces, i.e., the fact that \( \mathbf{U}_n^H (\mathbf{h}_1 \otimes \mathbf{s}_{1,1}) = \mathbf{0} \). In particular, we have

\[ \hat{\mathbf{h}}_1 = \arg \min_{\mathbf{h} \in \mathbb{C}^4} \| \mathbf{U}_n^H (\mathbf{h} \otimes \mathbf{s}_{1,1}) \|^2 \]

\[ = \arg \max_{\mathbf{h} \in \mathbb{C}^4} \| \mathbf{U}_s^H (\mathbf{h} \otimes \mathbf{s}_{1,1}) \|^2 \]

\[ = \arg \max_{\mathbf{h} \in \mathbb{C}^4} \left( \mathbf{h}^H \otimes \mathbf{s}_{1,1}^T \right) \mathbf{U}_s \mathbf{U}_s^H \left( \mathbf{h} \otimes \mathbf{s}_{11} \right) \]

\[ = \arg \max_{\mathbf{h} \in \mathbb{C}^4} \mathbf{h}^H \left[ \left( \mathbf{I}_4 \otimes \mathbf{s}_{1,1}^T \right) \mathbf{U}_s \mathbf{U}_s^H \left( \mathbf{I}_4 \otimes \mathbf{s}_{1,1} \right) \right] \mathbf{h} \]

\[ = \text{principle eigenvector of } \mathbf{Q}. \]  

(5.90)
In (5.91) $\hat{h}_1$ specifies $h_1$ up to an arbitrary complex scale factor $\alpha$, i.e. $\hat{h}_1 = \alpha h_1$. The following is the summary of a batch blind space-time multiuser detection algorithm for the two transmitter antenna/two receiver antenna configuration. The block length is $M$.

**Algorithm 1.** [Batch blind linear space-time multiuser detector – synchronous CDMA, two transmitter antennas and two receiver antennas]

- **Estimate the signal subspace:**
  \[
  \hat{C} = \frac{1}{M} \sum_{i=0}^{M-1} \hat{r}[i]\hat{r}[i]^H, \\
  = \hat{U}_s \hat{\Lambda}_s \hat{U}_s^H + \hat{U}_n \hat{\Lambda}_n \hat{U}_n^H. 
  \]

- **Estimate the channels:**
  \[
  \hat{Q}_1 = (I_4 \otimes s_{1,1}^T) \hat{U}_s \hat{U}_s^H (I_4 \otimes s_{1,1}), \\
  \hat{Q}_2 = (I_4 \otimes s_{2,1}^T) \hat{U}_s \hat{U}_s^H (I_4 \otimes s_{2,1}), \\
  \hat{h}_1 = \text{principal eigenvector of } \hat{Q}_1, \\
  \hat{h}_2 = \text{principal eigenvector of } \hat{Q}_2.
  \]

- **Form the detectors**
  \[
  \hat{w}_{1,1} = \hat{U}_s \hat{\Lambda}_s^{-1} \hat{U}_s^H (\hat{h}_1 \otimes s_{1,1}), \\
  \hat{w}_{2,1} = \hat{U}_s \hat{\Lambda}_s^{-1} \hat{U}_s^H (\hat{h}_2 \otimes s_{2,1}).
  \]
• Perform differential detection:

\[ z_{1,1}[i] = \mathbf{w}^{H}_{1,1}\tilde{\mathbf{r}}[i], \]
\[ z_{2,1}[i] = \mathbf{w}^{H}_{2,1}\tilde{\mathbf{r}}[i], \]
\[ \hat{\beta}_{1,1}[i] = \text{sign}\left(\Re\left\{ z_{1,1}[i]z_{1,1}[i - 1]^*\right\}\right), \]
\[ \hat{\beta}_{2,1}[i] = \text{sign}\left(\Re\left\{ z_{2,1}[i]z_{2,1}[i - 1]^*\right\}\right), \]
\[ i = 0, \cdots, M - 1. \]

A batch group-blind space-time multiuser detector algorithm can be implemented with simple modifications to (5.98) and (5.99).

b. Adaptive Blind Linear Space-Time Multiuser Detection

To form a sequential blind adaptive receiver, we need adaptive algorithms for sequentially estimating the channel and the signal subspace components \( \mathbf{U}_s \) and \( \mathbf{A}_s \). First, we address sequential adaptive channel estimation. Denote by \( \mathbf{z}[i] \) the projection of the stacked signal \( \tilde{\mathbf{r}}[i] \) onto the noise subspace, i.e.,

\[ \mathbf{z}[i] = \tilde{\mathbf{r}}[i] - U_s U_s^H \tilde{\mathbf{r}}[i] = U_n U_n^H \tilde{\mathbf{r}}[i]. \]

Since \( \mathbf{z}[i] \) lies in the noise subspace, it is orthogonal to any signal in the signal subspace, and in particular, it is orthogonal to \( (\mathbf{h}_1 \otimes \mathbf{s}_{1,1}) \). Hence \( \mathbf{h}_1 \) is the solution
to the following constrained optimization problem:

$$\min_{h_1 \in \mathbb{C}^4} E \left\{ \| z[i]^H (h_1 \otimes s_{1,1}) \|^2 \right\}$$

$$= \min_{h_1 \in \mathbb{C}^4} E \left\{ \| z[i]^H (I_4 \otimes s_{1,1}) h_1 \|^2 \right\}$$

$$= \min_{h_1 \in \mathbb{C}^4} E \left\{ \left\| \left( (I_4 \otimes s_{1,1}^T) z[i] \right)^H h_1 \right\|^2 \right\} \quad \text{s.t.} \quad \| h_1 \| = 1. \quad (5.106)$$

In order to obtain a sequential algorithm to solve the above optimization problem, we write it in the following (trivial) state space form

$$h_1[i + 1] = h_1[i], \quad \text{state equation}$$

$$0 = \left[ (I_4 \otimes s_{1,1}^T) z[i] \right]^H h_1[i], \quad \text{observation equation.}$$

The standard Kalman filter can then be applied to the above system as follows. Denote $$\hat{x}[i] \triangleq (I_4 \otimes s_{1,1}^T) z[i].$$

$$k[i] = \Sigma[i - 1] x[i] (x[i]^H \Sigma[i - 1] x[i])^{-1}, \quad (5.107)$$

$$h_1[i] = h_1[i - 1] - k[i] (x[i]^H h_1[i - 1]) / \| h_1[i - 1] - k[i] (x[i]^H h_1[i - 1]) \|, \quad (5.108)$$

$$\Sigma[i] = \Sigma[i - 1] - k[i] x[i]^H \Sigma[i - 1]. \quad (5.109)$$

Once we have obtained channel estimates at time slot $$i$$, we can combine them with estimates of the signal subspace components to form the detector in (5.84). Subspace tracking algorithms of various complexities exist in the literature. Since we are stacking received signal vectors and subspace tracking complexity increases at least linearly with signal subspace dimension, it is imperative that we choose an algorithm with minimal complexity. The best existing low-complexity algorithm for this purpose appears to be NAHJ-FST. This algorithm has the lowest complexity of
any algorithm used for similar purposes and has performed well when used for signal
subspace tracking in multipath fading environments. Since the size of \( \mathbf{U}_s \) is \( 4N \times 2K \),
the complexity is \( 40 \cdot 4N \cdot 2K + 3 \cdot 4N + 7.5(2K)^2 + 7 \cdot 2K \) floating operations per
iteration. The algorithm and a multiuser detection application is presented in [63].
The application to the current tracking problem is straightforward and will not be
discussed in detail.

**Algorithm 2.** [Blind adaptive linear space-time multiuser detector – synchronous
CDMA, two transmitter antennas and two receiver antennas]

- **Using a suitable signal subspace tracking algorithm, e.g. NAHJ-FST, update the signal
  subspace components \( \mathbf{U}_s[i] \) and \( \mathbf{A}_s[i] \) at each time slot \( i \).
- **Track the channel** \( \mathbf{h}_1[i] \) and \( \tilde{\mathbf{h}}_1[i] \) according to the following

\[
\begin{align*}
\mathbf{z}[i] &= \mathbf{r}[i] - \mathbf{U}_s[i]\mathbf{U}_s[i]^H \tilde{\mathbf{r}}[i], \\
\mathbf{x}[i] &= (\mathbf{I}_4 \otimes \mathbf{s}_{1,1}^T) \mathbf{z}[i], \\
\tilde{\mathbf{x}}[i] &= (\mathbf{I}_4 \otimes \mathbf{s}_{2,1}^T) \mathbf{z}[i], \\
\mathbf{k}[i] &= \mathbf{\Sigma}[i - 1] \mathbf{x}[i] (\mathbf{x}[i]^H \mathbf{\Sigma}[i - 1] \mathbf{x}[i])^{-1}, \\
\tilde{\mathbf{k}}[i] &= \tilde{\mathbf{\Sigma}}[i - 1] \tilde{\mathbf{x}}[i] (\tilde{\mathbf{x}}[i]^H \tilde{\mathbf{\Sigma}}[i - 1] \tilde{\mathbf{x}}[i])^{-1}, \\
\mathbf{h}_1[i] &= \mathbf{h}_1[i - 1] - \mathbf{k}[i] (\mathbf{x}[i]^H \mathbf{h}_1[i - 1]) / \\
& \quad \| \mathbf{h}_1[i - 1] - \mathbf{k}[i] (\mathbf{x}[i]^H \mathbf{h}_1[i - 1]) \|, \\
\tilde{\mathbf{h}}_1[i] &= \tilde{\mathbf{h}}_1[i - 1] - \tilde{\mathbf{k}}[i] (\tilde{\mathbf{x}}[i]^H \tilde{\mathbf{h}}_1[i - 1]) / \\
& \quad \| \tilde{\mathbf{h}}_1[i - 1] - \tilde{\mathbf{k}}[i] (\tilde{\mathbf{x}}[i]^H \tilde{\mathbf{h}}_1[i - 1]) \|, \\
\mathbf{\Sigma}[i] &= \mathbf{\Sigma}[i - 1] - \mathbf{k}[i] \mathbf{x}[i]^H \mathbf{\Sigma}[i - 1], \\
\tilde{\mathbf{\Sigma}}[i] &= \tilde{\mathbf{\Sigma}}[i - 1] - \tilde{\mathbf{k}}[i] \tilde{\mathbf{x}}[i]^H \tilde{\mathbf{\Sigma}}[i - 1].
\end{align*}
\]
• Form the detectors

\[
\mathbf{w}_{1,1}[i] = \mathbf{U}_s[i] A_s^{-1}[i] [i] \mathbf{U}_s[i]^H \left( \mathbf{h}_1[i] \otimes \mathbf{s}_{1,1} \right),
\]

(5.119)

\[
\mathbf{w}_{2,1}[i] = \mathbf{U}_s[i] A_s^{-1}[i] [i] \mathbf{U}_s[i]^H \left( \mathbf{h}_1[i] \otimes \mathbf{s}_{2,1} \right).
\]

(5.120)

• Perform differential detection:

\[
z_{1,1}[i] = \mathbf{w}_{1,1}[i]^H \mathbf{r}[i],
\]

(5.121)

\[
z_{2,1}[i] = \mathbf{w}_{2,1}[i]^H \mathbf{r}[i],
\]

(5.122)

\[
\hat{\beta}_{1,1}[i] = \text{sign} \left( \Re \left\{ z_{1,1}[i] z_{1,1}[i-1]^* \right\} \right),
\]

(5.123)

\[
\hat{\beta}_{2,1}[i] = \text{sign} \left( \Re \left\{ z_{2,1}[i] z_{2,1}[i-1]^* \right\} \right),
\]

(5.124)

A group-blind sequential adaptive space-time multiuser detector can be implemented similarly. The adaptive receiver structure is illustrated in Figure 12.

Fig. 12. Adaptive receiver structure for linear space-time multiuser detectors.
D. Blind Adaptive Space-Time Multiuser Detection for Asynchronous CDMA in Fading Multipath Channels

1. Signal Model

In this section, we develop adaptive space-time multiuser detectors for asynchronous CDMA systems with two transmitter and two receiver antennas. The continuous-time signal transmitted from antennas 1 and 2 due to the $k$-th user for time interval $i \in \{0, 1, \ldots\}$ is given by

$$x_k^{(1)}(t) = \sum_{i=0}^{M-1} \left[ b_{1,k}[i]s_{1,k}(t - 2iT_s) - b_{2,k}[i]s_{2,k}(t - (2i + 1)T_s) \right], \quad (5.125)$$

$$x_k^{(2)}(t) = \sum_{i=0}^{M-1} \left[ b_{2,k}[i]s_{2,k}(t - 2iT_s) + b_{1,k}[i]s_{1,k}(t - (2i + 1)T_s) \right] \quad (5.126)$$

where $M$ denotes the length of the data frame, $T_s$ denotes the information symbol interval, and $\{b_k[i]\}_i$ is the symbol stream of user $k$. Although this is an asynchronous system, we have, for notational simplicity, suppressed the delay associated with each users’ signal and incorporated it into the path delays in (5.128). We assume that for each $k$, the symbol stream, $\{b_k[i]\}_i$, is a collection of independent random variables that take on values of +1 and −1 with equal probability. Furthermore, we assume that the symbol streams of different users are independent. For the direct-sequence spread-spectrum (DS-SS) format, the user signaling waveforms have the form

$$s_{q,k}(t) = \sum_{j=0}^{N-1} c_{q,k}[j]\psi(t - jT_c), \quad 0 \leq t \leq T, \quad (5.127)$$

where $N$ is the processing gain, $\{c_{q,k}[j]\}_i, q \in \{1, 2\}$ is a signature sequence of ±1’s assigned to the $k$-th user for bit $b_{q,k}[i]$, and $\psi(t)$ is a normalized chip waveform of duration $T_c = T_s/N$. The $k$-th user’s signals, $x_k^{(1)}(t)$ and $x_k^{(2)}(t)$, propagate from transmitter antenna $a$ to receiver antenna $b$ through a multipath fading channel whose
impulse response is given by

\[ g_k^{(a,b)}(t) = \sum_{l=1}^{L} \alpha_{kl}^{(a,b)} \delta \left( t - \tau_{kl}^{(a,b)} \right), \quad (5.128) \]

where \( \alpha_{kl}^{(a,b)} \) is the complex path gain from antenna \( a \) to antenna \( b \) associated with the \( l \)-th path for the \( k \)-th user, and \( \tau_{kl}^{(a,b)} \), \( \tau_{k1}^{(a,b)} < \tau_{k2}^{(a,b)} < \cdots < \tau_{kL}^{(a,b)} \) is the sum of the corresponding path delay and the initial transmission delay of user \( k \). It is assumed that the channel is slowly varying, so that the path gains and delays remain constant over the duration of one signal frame \((MT_s)\).

The received signal component due to the transmission of \( x_k^{(1)}(t) \) and \( x_k^{(2)}(t) \) through the channel at receiver antennas 1 and 2 is given by

\[
\begin{align*}
y_k^{(1)}(t) &= x_k^{(1)}(t) \ast g_k^{(1,1)}(t) + x_k^{(2)}(t) \ast g_k^{(2,1)}(t), \quad (5.129) \\
y_k^{(2)}(t) &= x_k^{(1)}(t) \ast g_k^{(1,2)}(t) + x_k^{(2)}(t) \ast g_k^{(2,2)}(t). \quad (5.130)
\end{align*}
\]

Substituting (5.125-5.126) and (5.128) into (5.129-5.130) we have for receiver antenna \( b \in \{1, 2\} \)

\[
\begin{align*}
y_k^{(b)}(t) &= \sum_{i=0}^{M-1} \left[ b_{1,k}[i]s_{1,k}(t - 2iT_s) \ast g_k^{(1,b)}(t) - b_{2,k}[i]s_{2,k}(t - (2i + 1)T_s) \ast g_k^{(1,b)}(t) \right] + \\
&\quad \sum_{i=0}^{M-1} \left[ b_{2,k}[i]s_{2,k}(t - 2iT_s) \ast g_k^{(2,b)}(t) + b_{1,k}[i]s_{1,k}(t - (2i + 1)T_s) \ast g_k^{(2,b)}(t) \right].
\end{align*}
\]
For $a, b, q \in \{1, 2\}$ we define

$$h_{q,k}^{(a,b)}(t) \triangleq s_{q,k}(t) \ast g_{k}^{(a,b)}(t) = \sum_{j=0}^{N-1} c_{q,k}[j] \left[ \sum_{l=1}^{L} \alpha_{kl}^{(a,b)} \psi \left( t - jT_{c} - \tau_{kl}^{(a,b)} \right) \right].$$

(5.131)

In (5.131), $g_{k}^{(a,b)}(t)$ is the composite channel response for the channel between transmitter antenna $a$ and receiver antenna $b$, taking into account the effects of the chip pulse waveform and the multipath channel. Then we have

$$y_{k}^{(b)}(t) = \sum_{i=0}^{M-1} \left[ b_{1,k}[i] h_{1,k}^{(1,b)}(t - 2iT_{s}) - b_{2,k}[i] h_{2,k}^{(1,b)}(t - (2i + 1)T_{s}) \right] + \sum_{i=0}^{M-1} \left[ b_{2,k}[i] h_{2,k}^{(2,b)}(t - 2iT_{s}) + b_{1,k}[i] h_{1,k}^{(2,b)}(t - (2i + 1)T_{s}) \right].$$

(5.132)

The total received signal at receiver antenna $b \in \{1, 2\}$ is given by

$$r^{(b)}(t) = \sum_{k=1}^{K} y_{k}^{(b)}(t) + v^{(b)}(t).$$

(5.133)

At the receiver, the received signal is match filtered to the chip waveform and sampled at the chip rate, i.e., the sampling interval is $T_{c}$, $N$ is the total number of samples per symbol interval, and $2N$ is the total number of samples per time slot. The $n$-th matched filter output during the $i$-th time slot is given by
\( r^{(b)}[i, n] = \int_{2iT_s + nT_c}^{2iT_s + (n+1)T_c} r^{(b)}(t) \psi(t - 2iT_s - nT_c) dt \)
\[ = \sum_{k=1}^{K} \left\{ \int_{2iT_s + nT_c}^{2iT_s + (n+1)T_c} \psi(t - 2iT_s - nT_c) y_k^{(b)}(t) dt \right\} + \int_{2iT_s + nT_c}^{2iT_s + (n+1)T_c} v^{(b)}(t) \psi(t - 2iT_s - nT_c) dt. \]
\( v^{(b)}[i, n] \) \( y_k^{(b)}[i, n] \)

Denote the maximum delay (in symbol intervals) as
\( \tau^{(a,b)}_k \triangleq \left\lfloor \frac{\tau^{(a,b)}_k L + T_c}{T_s} \right\rfloor \) and \( \tau \triangleq \max_{k,a,b} \tau^{(a,b)}_k. \)

Substituting (5.132) into (5.134) we obtain
\[ y^{(b)}_k[i, n] = \sum_{p=0}^{M-1} \left\{ b_{1,k}[p] \int_{2iT_s + nT_c}^{2iT_s + (n+1)T_c} h^{(1,b)}_{1,k}(t - 2pT_s) \psi(t - 2iT_s - nT_c) dt - b_{2,k}[p] \int_{2iT_s + nT_c}^{2iT_s + (n+1)T_c} h^{(1,b)}_{2,k}(t - (2p + 1)T_s) \psi(t - 2iT_s - nT_c) dt + b_{2,k}[p] \int_{2iT_s + nT_c}^{2iT_s + (n+1)T_c} h^{(2,b)}_{2,k}(t - 2pT_s) \psi(t - 2iT_s - nT_c) dt + b_{1,k}[p] \int_{2iT_s + nT_c}^{2iT_s + (n+1)T_c} h^{(2,b)}_{1,k}(t - (2p - 1)T_s) \psi(t - 2iT_s - nT_c) dt \right\}. \]
Further substitution of (5.131) into (5.136) shows that

\[ y_k^{(b)}[i, n] = \sum_{p=0}^{M-1} \left\{ b_{1,k}[p] \sum_{j=0}^{N-1} c_{1,k}[j] \sum_{l=1}^{L} \alpha_{kl}^{(1,b)} \int_{2iT + (n+1)T}^{2iT + nT} \psi(t - 2iT - nT) \psi(t - 2pT - jT - \tau_{kl}^{(1,b)}) dt - \right. \]

\[ \left. b_{2,k}[p] \sum_{j=0}^{N-1} c_{2,k}[j] \sum_{l=1}^{L} \alpha_{kl}^{(1,b)} \int_{2iT + (n+1)T}^{2iT + nT} \psi(t - 2iT - nT) \psi(t - (2p + 1)T - jT - \tau_{kl}^{(1,b)}) dt + \right. \]

\[ \left. b_{2,k}[p] \sum_{j=0}^{N-1} c_{2,k}[j] \sum_{l=1}^{L} \alpha_{kl}^{(2,b)} \int_{2iT + (n+1)T}^{2iT + nT} \psi(t - 2iT - nT) \psi(t - 2pT - jT - \tau_{kl}^{(2,b)}) dt + \right. \]

\[ \left. b_{1,k}[p] \sum_{j=0}^{N-1} c_{1,k}[j] \sum_{l=1}^{L} \alpha_{kl}^{(2,b)} \int_{2iT + (n+1)T}^{2iT + nT} \psi(t - 2iT - nT) \psi(t - (2p + 1)T - jT - \tau_{kl}^{(2,b)}) dt \right\} \]

\[ = \sum_{p=0}^{[s/2] - 1} \left\{ b_{1,k}[i - p] \sum_{j=0}^{N-1} c_{1,k}[j] \sum_{l=1}^{L} \alpha_{kl}^{(1,b)} \int_{0}^{T} \psi(t) \psi(t - jT - \tau_{kl}^{(1,b)}) dt + \right. \]

\[ \left. f_k^{(1,b)}[n + 2pN - j] \right\} - \sum_{p=0}^{[s/2] - 1} \left\{ b_{2,k}[i - p] \sum_{j=0}^{N-1} c_{2,k}[j] \sum_{l=1}^{L} \alpha_{kl}^{(1,b)} \int_{0}^{T} \psi(t) \psi(t - jT - \tau_{kl}^{(1,b)}) dt + \right. \]

\[ \left. f_k^{(1,b)}[n + 2pN - N - j] \right\} + \sum_{p=0}^{[s/2] - 1} \left\{ b_{2,k}[i - p] \sum_{j=0}^{N-1} c_{2,k}[j] \sum_{l=1}^{L} \alpha_{kl}^{(2,b)} \int_{0}^{T} \psi(t) \psi(t - jT - \tau_{kl}^{(2,b)}) dt + \right. \]

\[ \left. f_k^{(2,b)}[n + 2pN - j] \right\} - \sum_{p=0}^{[s/2] - 1} \left\{ b_{2,k}[i - p] \sum_{j=0}^{N-1} c_{2,k}[j] \sum_{l=1}^{L} \alpha_{kl}^{(2,b)} \int_{0}^{T} \psi(t) \psi(t - jT - \tau_{kl}^{(2,b)}) dt + \right. \]

\[ \left. f_k^{(2,b)}[n + 2pN - N - j] \right\} + \sum_{p=0}^{[s/2] - 1} \left\{ b_{1,k}[i - p] \sum_{j=0}^{N-1} c_{1,k}[j] \sum_{l=1}^{L} \alpha_{kl}^{(2,b)} \int_{0}^{T} \psi(t) \psi(t - jT - \tau_{kl}^{(2,b)}) dt + \right. \]

\[ \left. f_k^{(2,b)}[n + 2pN - N - j] \right\} - \sum_{p=0}^{[s/2] - 1} \left\{ b_{1,k}[i - p] \sum_{j=0}^{N-1} c_{1,k}[j] \sum_{l=1}^{L} \alpha_{kl}^{(2,b)} \int_{0}^{T} \psi(t) \psi(t - jT - \tau_{kl}^{(2,b)}) dt \right\},(5.137) \]
We may write \( y^{(b)}_{k}[i, n] \) more compactly as

\[
y^{(b)}_{k}[i, n] = \sum_{j=0}^{[i/2]} \left( h^{(1,b)}_{1,k}[j, n]b_{1,k}[i - j] - h^{(1,b)}_{2,k}[j, n]b_{2,k}[i - j] + h^{(2,b)}_{2,k}[j, n]b_{2,k}[i - j] + h^{(2,b)}_{1,k}[j, n]b_{1,k}[i - j] \right)
\]

\[
= \sum_{j=0}^{[i/2]} b_{1,k}[i - j]g^{(b)}_{1,k}[j, n] + \sum_{j=0}^{[i/2]} b_{2,k}[i - j]g^{(b)}_{2,k}[j, n], \tag{5.138}
\]

where

\[
g^{(b)}_{1,k}[j, n] \triangleq h^{(1,b)}_{1,k}[j, n] + h^{(2,b)}_{1,k}[j, n] \tag{5.139}
\]

\[
g^{(b)}_{2,k}[j, n] \triangleq h^{(2,b)}_{2,k}[j, n] - h^{(1,b)}_{2,k}[j, n]. \tag{5.140}
\]

For \( j = 0, 1, \ldots, [i/2] \) denote

\[
\frac{H^{(b)}[j]}{2N \times 2K} \triangleq \begin{bmatrix}
g^{(b)}_{1,1}[j, 0] & \cdots & g^{(b)}_{1,K}[j, 0] & g^{(b)}_{2,1}[j, 0] & \cdots & g^{(b)}_{2,K}[j, 0] \\
\vdots & \vdots & \vdots & \vdots & & \vdots \\
g^{(b)}_{1,1}[j, 2N - 1] & \cdots & g^{(b)}_{1,K}[j, 2N - 1] & g^{(b)}_{2,1}[j, 2N - 1] & \cdots & g^{(b)}_{2,K}[j, 2N - 1]
\end{bmatrix},
\]

\[
x^{(b)}[i] \triangleq \begin{bmatrix}
x^{(b)}[i, 0] \\
\vdots \\
x^{(b)}[i, 2N - 1]
\end{bmatrix}_{2N \times 1}, \quad v^{(b)}[i] \triangleq \begin{bmatrix}
v^{(b)}[i, 0] \\
\vdots \\
v^{(b)}[i, 2N - 1]
\end{bmatrix}_{2N \times 1}, \quad b[i] \triangleq \begin{bmatrix}
b_{1,1}[i] \\
\vdots \\
b_{1,K}[i] \\
b_{2,1}[i] \\
\vdots \\
b_{2,K}[i]
\end{bmatrix}_{2K \times 1}, \tag{5.141}
\]

Then we have

\[
x^{(b)}[i] = \sum_{j=0}^{[i/2]} \frac{H^{(b)}[j]b[i - j] + v^{(b)}[i]}{H^{(b)}[i] + b[i]}. \tag{5.142}
\]
To exploit both time and spatial diversity, we stack the vectors received from both receive antennas,

\[
\mathbf{r}[i] \triangleq \begin{bmatrix} r^{(1)}[i] \\ r^{(2)}[i] \end{bmatrix}_{4N \times 1}, \tag{5.143}
\]

and observe that

\[
\mathbf{r}[i] = \mathbf{H}[i] \ast \mathbf{b}[i] + \mathbf{v}[i], \tag{5.144}
\]

where

\[
\mathbf{H}[j] \triangleq \begin{bmatrix} H^{(1)}[j] \\ H^{(2)}[j] \end{bmatrix}_{4N \times 2K}, \quad j = 0, 1, \ldots, \lfloor \nu/2 \rfloor \quad \text{and} \quad \mathbf{v}[i] \triangleq \begin{bmatrix} v^{(1)}[i] \\ v^{(2)}[i] \end{bmatrix}_{4N \times 1}. \tag{5.145}
\]

By stacking \( m \) successive received sample vectors, we create the following quantities:

\[
\mathbf{r}[i] \triangleq \begin{bmatrix} r[i] \\ \vdots \\ r[i + m - 1] \end{bmatrix}_{4Nm \times 1}, \quad \mathbf{v}[i] \triangleq \begin{bmatrix} v[i] \\ \vdots \\ v[i + m - 1] \end{bmatrix}_{4Nm \times 1}, \quad \mathbf{b}[i] \triangleq \begin{bmatrix} b[i - \lfloor \nu/2 \rfloor] \\ \vdots \\ b[i + m - 1] \end{bmatrix}_{r \times 1},
\]

\[
\mathbf{H} \triangleq \begin{bmatrix} H[\lfloor \nu/2 \rfloor] & \cdots & H[0] & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & H[\lfloor \nu/2 \rfloor] & \cdots & H[0] \end{bmatrix}_{4Nm \times r}, \tag{5.146}
\]

where \( r \triangleq 2K(m + \lfloor \nu/2 \rfloor) \). We can write (5.144) in matrix form as

\[
\mathbf{r}[i] = \mathbf{H} \mathbf{b}[i] + \mathbf{v}[i]. \tag{5.147}
\]

We will see in Section b that the smoothing factor, \( m \), is chosen such that

\[
m \geq \left\lceil \frac{N(\nu + 1) + K \lfloor \nu/2 \rfloor + 1}{2N - K} \right\rceil \tag{5.148}
\]
for channel identifiability. Note that the columns of $H$ (the composite signature vectors) contain information about both the timings and the complex path gains of the multipath channel of each user. Hence an estimate of these waveforms eliminates the need for separate estimates of the timing information $\{r_{kl}^{(a,b)}\}_{l=1}^L$.

2. Blind MMSE Space-Time Multiuser Detection

Since the ambient noise is white, i.e., $E\{v[i]v[i]^H\} = \sigma^2 I_{4Nm}$, the autocorrelation matrix of the received signal in (5.147) is

$$R \triangleq E\{r[i]r[i]^H\} = HH^H + \sigma^2 I_{4Nm}$$ (5.149)

$$= U_sA_sU_s^H + \sigma^2 U_nU_n^H,$$ (5.150)

where (5.150) is the eigendecomposition of $R$. $U_s$ has size $4Nm \times r$ and $U_n$ has size $4Nm \times (4Nm - r)$.

The MMSE space-time multiuser detector and corresponding bit estimate for $b_{a,k}[i], a \in \{1, 2\}$ are given by

$$w_{a,k}[i] \triangleq \arg \min_{w \in \mathbb{C}^{4Nm}} E\{|b_{a,k}[i] - w^Hr[i]|^2\},$$ (5.151)

$$\hat{b}_{a,k}[i] = \text{sign} \left( \text{Re}\left\{ w_{a,k}[i]^H r[i] \right\} \right).$$ (5.152)

The solution to (6.19) can be written in terms of the signal subspace components as [11]

$$w_{a,k}[i] = U_sA_s^{-1}U_s^H h_{a,k},$$ (5.153)

where $h_{a,k} \triangleq He_K(2a+1)+a-1+k$ is the composite signature waveform of user $k$ for bit $a \in \{1, 2\}$. This detector is termed blind since it requires knowledge only of the signature sequence of the user of interest. Of course, we also require estimates of
the signal subspace components and of the channel. We address the issue of channel estimation next.

3. Blind Adaptive Channel Estimation

In this section we extend the blind adaptive channel estimation technique described in Section b to the asynchronous multipath case. First, however, we describe the discrete-time channel model in order to formulate an analog to the optimization problem in (5.106).

a. Discrete-Time Channel Model

Using (5.141) and (5.145) it is easy to see that

\[
\mathbf{h}_{a,k} = \begin{bmatrix}
g_{a,k}^{(1)}[0, 0] \\
g_{a,k}^{(1)}[0, 2N - 1] \\
\vdots \\
g_{a,k}^{(1)}[[t/2], 0] \\
g_{a,k}^{(1)}[[t/2], 2N - 1] \\
g_{a,k}^{(2)}[0, 0] \\
\vdots \\
g_{a,k}^{(2)}[0, 2N - 1] \\
g_{a,k}^{(2)}[[t/2], 0] \\
g_{a,k}^{(2)}[[t/2], 2N - 1]
\end{bmatrix}_{4N([t/2]+1) \times 1}.
\]
From (5.138) we have for $j = 0, \ldots, [\ell / 2]; n = 0, \ldots, 2N - 1; b = 1, 2$

\begin{align*}
g^{(b)}_{1,k}[j, n] &= h^{(1,b)}_{1,k}[j, n] + h^{(2,b)}_{1,k}[j, n] \quad (5.155) \\
g^{(b)}_{2,k}[j, n] &= h^{(2,b)}_{2,k}[j, n] - h^{(1,b)}_{2,k}[j, n]. \quad (5.156)
\end{align*}

We will develop the discrete-time channel model for $g^{b}_{1,k}[j, n]$. The development for $g^{b}_{2,k}[j, n]$ follows similarly. From (5.137) we see that

\[ g^{b}_{1,k}[j, n] = \sum_{q=0}^{N-1} c_{1,k}[q] f^{(1,b)}_{k}[n + 2jN - q] + \sum_{q=0}^{N-1} c_{1,k}[q] f^{(2,b)}_{k}[n + 2jN - N - q]. \quad (5.157) \]

From (5.137) we can also see that the sequences $f^{1,b}_{k}[i]$ and $f^{2,b}_{k}[i]$ are zero whenever $i < 0$ or $i > \ell N$. With this in mind we define the following vectors

\begin{align*}
\mathbf{g}^{(b)}_{1,k} &\triangleq \begin{bmatrix} g^{(b)}_{1,k}[0, 0] & \cdots & g^{(b)}_{1,k}[0, 2N - 1] & \cdots & g^{(b)}_{1,k}[[\ell / 2], 0] & \cdots & g^{(b)}_{1,k}[[\ell / 2], 2N - 1] \end{bmatrix}^T, \quad (5.158) \\
\mathbf{f}^{(1,b)}_{1,k} &\triangleq \begin{bmatrix} f^{(1,b)}_{k}[0] & \cdots & f^{(1,b)}_{k}[\ell N] \end{bmatrix}^T, \quad (5.159) \\
\mathbf{f}^{(2,b)}_{1,k} &\triangleq \begin{bmatrix} 0 \cdots 0 & f^{(2,b)}_{k}[0] & \cdots & f^{(2,b)}_{k}[\ell N] \end{bmatrix}^T. \quad (5.160)
\end{align*}

Then (5.158) can be written as

\[ \mathbf{g}^{(b)}_{1,k} = \mathbf{C}_{1,k} \begin{bmatrix} \mathbf{f}^{(1,b)}_{1,k} + \mathbf{f}^{(2,b)}_{1,k} \\ \mathbf{f}^{(b)}_{1,k} \end{bmatrix} \quad (5.161) \]
where

\[
C_{1,k} \triangleq \begin{bmatrix}
  c_{1,k}[0] \\
  c_{1,k}[1] & \ddots & \\
  \vdots & \ddots & c_{1,k}[0] \\
  \vdots & & c_{1,k}[1] \\
  c_{1,k}[N-1] & & \vdots \\
  \cdots & & \cdots \\
  c_{1,k}[N-1]
\end{bmatrix}^{(2N([t/2]+1)) \times (N(t+1)+1)}
\] (5.162)

A similar development for \( g_{2,k}^b[j, n] \) produces the result

\[
g_{2,k}^b = C_{2,k} \cdot \underbrace{f_{2,k}^{(2,b)} - f_{2,k}^{(1,b)}}_{f_{2,k}^b},
\] (5.163)

where

\[
f_{2,k}^{(2,b)} \triangleq \begin{bmatrix}
  f_k^{(2,b)}[0] \cdots f_k^{(2,b)}[tN] \\
  \vdots & & \vdots \\
  0 \cdots 0
\end{bmatrix}_N \text{zeros}^T
\] (5.164)

\[
f_{2,k}^{(1,b)} \triangleq \begin{bmatrix}
  0 \cdots 0 \\
  \vdots & & \vdots \\
  f_k^{(1,b)}[0] \cdots f_k^{(1,b)}[tN] \\
\end{bmatrix}_N \text{zeros}^T.
\] (5.165)

The final task in the section is to form expressions for the composite signature waveforms \( h_{1,k} \) and \( h_{2,k} \) in terms of the signature matrices \( C_{1,k}, C_{2,k} \) and the channel response vectors \( f_{1,k}^b \) and \( f_{2,k}^b \). Denote by \( C_{a,k}^j[j], j = 0, 1, \ldots, \lceil t/2 \rceil, a \in \{1, 2\} \) the submatrix of \( C_{a,k} \) consisting of rows \( 2Nj + 1 \) through \( 2(j + 1)N \). Then it is easy to show that

\[
h_{a,k} = \overline{C}_{a,k} f_{a,k}
\] (5.166)
where

$$
\overline{C}_{a,k} \triangleq \begin{bmatrix}
C_{a,k}[0] & 0 \\
0 & C_{a,k}[0] \\
C_{a,k}[1] & 0 \\
0 & C_{a,k}[1] \\
\vdots & \vdots \\
C_{a,k}[[l/2]] & 0 \\
0 & C_{a,k}[[l/2]]
\end{bmatrix}^{4N([l/2]+1)\times(2N(l+1)+2)}
$$

and

$$
f_{a,k} \triangleq \begin{bmatrix}
f_{a,k}^{(1)} \\
f_{a,k}^{(2)}
\end{bmatrix}^{(2N(l+1)+2)\times1}
$$

b. Blind Sequential Kalman Channel Estimation

The blind channel estimation problem for the asynchronous multipath case involves the estimation of $f_{a,k}$ ($1 \leq k \leq K$, $a = 1, 2$) from the received signal $r[i]$. As we did for the synchronous case, we will exploit the orthogonality between the signal subspace and noise subspace. Specifically, since $U_n$ is orthogonal to the columnspace of $H$, we have

$$
U_n^H h_{a,k} = U_n^H \overline{C}_{a,k} f_{a,k} = 0.
$$

Denote by $z[i]$ the projection of the received signal $r[i]$ onto the noise subspace, i.e.,

$$
z[i] = r[i] - U_s U_s^H r[i]
$$

Using (5.168) we have

$$
f_{a,k}^H \overline{C}_{a,k}^H z[i] = 0.
$$
Our channel estimation problem, then, involves the solution of the optimization problem

\[
\hat{f}_{a,k} = \arg \min_{f} E \left\{ |f H C_{a,k} z[i]|^2 \right\} \tag{5.172}
\]

subject to the constraint \( \|f\| = 1 \). If we denote \( x[i] \triangleq C_{a,k}^{H} z[i] \) then we can use the Kalman-type algorithm described in (5.107)-(5.109) where \( h_1[i] \) is replaced with \( f_{a,k[i]} \).

Note that a necessary condition for the channel estimate to be unique is that the matrix \( U_{n}^{H} C_{a,k} \) is tall, i.e. \( 4Nm - 2K(m + \lceil \iota/2 \rceil) \geq 2N(\iota + 1) + 2 \). Therefore we choose the smoothing factor, \( m \), such that

\[
m \geq \left\lfloor \frac{N(\iota + 1) + K\lceil \iota/2 \rceil + 1}{2N - K} \right\rfloor. \tag{5.173}
\]

Using the same constraint, we find that for a fixed \( m \), the maximum number of users that can be supported is

\[
\min \left\{ \left\lfloor \frac{N(2m - \iota - 1) - 1}{m + \lceil \iota/2 \rceil} \right\rfloor, \left\lfloor \frac{N}{2} \right\rfloor \right\}. \tag{5.174}
\]

Notice that for reasonable choices of \( m \) and \( \iota \), (5.174) is larger than the maximum number of users for the linear diversity receiver structure, given by

\[
\left\lfloor \frac{N(m - \iota)}{2(m + \iota)} \right\rfloor. \tag{5.175}
\]

This represents a quantitative example of the capacity benefit of space-time multiuser detection discussed in Section 4.

Once an estimate of the channel state, \( \hat{f}_{a,k} \), is obtained, the composite signature vector of the \( k \)-th user for bit \( a \) is given by (5.166). Note that there is an arbitrary phase ambiguity in the estimated channel state, which necessitates differential encoding and decoding of the transmitted data.
4. Algorithm Summary

**Algorithm 3.** [Blind adaptive linear space-time multiuser detector – asynchronous multipath CDMA, two transmitter antennas and two receiver antennas]

- Stack matched filter outputs in (5.134) according to (5.141), (5.143), and (5.146) to create $r[i]$.
- Create $C_{a,k}$ according to (5.167).
- Using a suitable signal subspace tracking algorithm, e.g. NAHJ-FST, update the signal subspace components $U_s[i]$ and $A_s[i]$ at each time slot $i$.
- Track the channel $f_{a,k}$ ($1 \leq k \leq K, a = 1, 2$) according to the following
  \[
  z[i] = r[i] - U_s[i]U_s[i]^H r[i], \quad (5.176)
  \]
  \[
  x[i] = C_{a,k}^H z[i], \quad (5.177)
  \]
  \[
  k[i] = \Sigma[i - 1] x[i] (x[i]^H \Sigma[i - 1] x[i])^{-1}, \quad (5.178)
  \]
  \[
  f_{a,k}[i] = f_{a,k}[i - 1] - k[i] (x[i]^H f_{a,k}[i - 1]) / \|f_{a,k}[i - 1] - k[i] x[i]^H f_{a,k}[i - 1]\|, \quad (5.179)
  \]
  \[
  \Sigma[i] = \Sigma[i - 1] - k[i] x[i]^H \Sigma[i - 1], \quad (5.180)
  \]
- Form the detectors
  \[
  w_{a,k}[i] = U_s[i] \Lambda_s^{-1}[i] U_s[i]^H C_{a,k} f_{a,k}[i]. \quad (5.181)
  \]
- Perform differential detection:
  \[
  z_{a,k}[i] = w_{a,k}[i]^H r[i], \quad (5.182)
  \]
  \[
  \hat{\beta}_{a,k}[i] = \text{sign} \left( \Re \left\{ z_{a,k}[i] z_{a,k}[i - 1]^* \right\} \right). \quad (5.183)
  \]
E. Simulation Results

In this section, we present simulation results to illustrate the performance of blind adaptive space-time multiuser detection. We first look at the synchronous flat-fading case; then we consider the asynchronous multipath-fading scenario. For all simulations we use the two transmit/two receive antenna configuration. The \(m\)-sequences of length 15 and their shifted versions are employed as user spreading sequences. The chip pulse is a raised cosine with roll-off factor \(0.5\). For the multipath case, each user has \(L = 3\) paths. The delay of each path is uniform on \([0, T_s]\). Hence, the maximum delay spread is one symbol interval, i.e., \(\tau = 1\). The fading gain for each users’ channel is generated from a complex Gaussian distribution and is fixed for all simulations. The path gains in each users’s channel are normalized so that each users’s signal arrives at the receiver with the same power. The smoothing factor is \(m = 2\) and the forgetting factor for the subspace tracking algorithm for all simulations is \(0.995\). The performance measures are bit-error probability and signal-to-interference-plus-noise ratio, defined by \(\text{SINR} \triangleq \frac{E^2\{w^H r\}}{\text{Var}\{w^H r\}}\), where the expectation is with respect to the data bits of interfering users and the ambient noise. In the simulations, the expectation operation is replaced by the time averaging operation. SINR is a particularly appropriate figure of merit for MMSE detectors since it has been shown [21] that the output of an MMSE detector is approximately Gaussian distributed. Hence, the SINR values translate directly and simply to bit-error probabilities, i.e., \(Pr(e) \approx Q(\sqrt{\text{SINR}})\). The labeled horizontal lines on the SINR plot represent bit-error-probability thresholds. For the SINR plots, the number of users for the first 1500 iterations is 4. At iteration 1501, 3 users are added so that the system is fully loaded. At iteration 3001, 5 users are removed. For the BER plots, the frame size is 200 bits and the system is allowed 1000 bits to reach steady state before errors are
accumulated.

Figure 13 illustrates the adaptation performance for the synchronous, flat fading case. The SNR is fixed at 8dB. Notice that the bit-error-probability does not drop below $10^{-3}$ even during transitions when users enter or leave the system.

Figure 14 shows the steady state performance for the synchronous case for different system loads. We see that the performance changes little as the system load changes. Although an error floor is unavoidable since we are estimating the detectors and the channel from the received signal, it is sufficiently low so that it does not appear in this figure.

Figure 15 shows the adaptation performance for the asynchronous multipath case. The SNR for this simulation is 11dB. Again, notice that the bit-error-probability does not drop significantly as users enter and leave the system.

Figure 16 shows the steady state performance for the asynchronous multipath case for different system loads. It is seen that system loading has a more significant effect on performance for the asynchronous multipath case that it does for the synchronous case.

F. Conclusion

In this chapter, we have analyzed and compared two different linear receiver structures that are appropriate for CDMA systems with multiple transmit and receive antennas. We have seen that the space-time structure has many advantages over linear diversity combining, including better bit-error-rate performance (for configurations with 1 transmit antenna and 2 or more receive antennas), lower complexity, and higher user capacity. We have also developed blind adaptive implementations of the space-time structure for synchronous CDMA channels in flat fading channels and
Fig. 13. Adaptation performance of space-time multiuser detection for synchronous CDMA. The labelled horizontal lines represent bit-error-probability thresholds.
Fig. 14. Steady state performance of space-time multiuser detection for synchronous CDMA.
Fig. 15. Adaptation performance of space-time multiuser detection for asynchronous multipath CDMA. The labelled horizontal lines represent bit-error-probability thresholds.
Fig. 16. Steady state performance of space-time multiuser detection for asynchronous multipath CDMA.
for asynchronous CDMA in fading multipath channels. Finally, we have presented simulations to illustrate the steady state and adaptation performance of the adaptive receiver.
CHAPTER VI

ADAPTIVE TRANSMITTER OPTIMIZATION FOR BLIND AND GROUP-BLIND MULTIUSER DETECTION

The interest in high-rate wireless services such as data and video means that future generation code-division multiple-access (CDMA) systems must be able to cope efficiently with heterogeneous traffic. In order for a system to efficiently serve different traffic types in a dynamic environment, it is necessary that the design should involve adaptive optimization of the receiver and the transmitter. Typical physical-layer work in adaptive multiuser detection for CDMA considers the transmitter parameters (rate, power, spreading codes, error-correction codes, spreading gain) to be fixed. Optimization is usually attempted at the receiver only. In recent years, more researchers have investigated transmitter optimization, but usually in the context of rate optimization or power control. Other transmitter optimization work includes joint rate and power control as considered in [64, 65, 66, 67]. Joint power control and transmit beamforming are investigated in [68, 69, 70]. Joint power control and receiver filter coefficient optimization is considered in [71, 72, 73]. Other work in transmitter optimization includes the optimization of the transmitter bandwidth and spectral density in [74] and the MMSE decision-feedback equalization work in [75].

The practicality of these works is limited, however, since the channel is either static, perfectly-known, or simply AWGN. There is very little work in adaptive systems for CDMA that utilize some form of transmitter optimization with multiuser detection.

One of the CDMA transmitter parameters that is largely ignored in adaptive systems is the spreading code. It is well known that CDMA systems are interference limited. It has been shown recently in [12] that practical systems, where multiuser detectors must be estimated, are also estimation-error limited, i.e., performance is
limited by the difference between the (unavailable) exact detector and the (available) estimated detector. Multiuser detection performance in estimation/interference limited environments improves when the correlation of the spreading codes decreases. With multiple-access interference reduction in mind, researchers have considered optimal (binary) spreading sequences for synchronous CDMA over AWGN channels when the number of users is larger than the spreading gain [76]. They have also identified good spreading sequences in the context of a spread-spectrum systems with conventional matched filter receivers and equal received power for all users [77]. In [78] the authors addressed the problem of code sequence design in an information-theoretic setting for which the sum of the rates of all users is maximized. In [79], Gold designed codes with good correlation properties that work well in both synchronous and asynchronous environments. Despite the significant body of work in binary spreading sequence design, however, very little work has been done on adapting spreading codes using information about the channel for fading multipath environments. This is a relevant problem since spreading code sets that have good correlation properties may lead to composite signature waveform sets (non-binary sets composed of the convolution of the channel with the spreading codes) that have poor correlation properties. Good composite signature waveform sets can only be constructed with information about the channel. This is essentially a problem of dynamic channel allocation. One of the contributions of the present work is a simple blind (for downlink scenarios) or group-blind (for uplink scenarios) algorithm for adapting user spreading codes in asynchronous fading multipath environments.

Power is the transmitter parameter that is most often exploited to improve performance in CDMA systems. This is due, in part, to the so-called near-far problem, in which correlation among users’ spreading codes (or composite signature waveforms) can cause severe performance discrepancies between transmitters that are close to the
base station and those that are distant from the base station when transmitter power is unregulated. Initially, the goal of power control was simply to regulate transmitter power to maintain a minimum system wide performance criteria \cite{80}, typically measured in signal-to-carrier power ratio or signal-to-interference-plus-noise ratio (SINR).

The satisfaction that a user received in such a system, i.e., the utility, was a binary function that was zero when the SINR dropped below a threshold and unity when the SINR achieved or surpassed the threshold. This is appropriate for voice communications in which SINRs above a threshold do not provide additional benefit and SINRs below that threshold lead to unintelligible speech which has zero benefit (utility) for the user. However, this kind of utility function is not appropriate for data because data services must meet different requirements to satisfy the user. In particular, data applications are typically delay tolerant, but require very low bit-error-rates. In the utility-pricing work of \cite{81}, the authors develop a utility function for data that explicitly takes these requirements into account. This function has units of “benefit per cost” in bits per Joule. Although heuristically pleasing, there does not currently exist a convenient algorithm for implementing this scheme in practice because of the high complexity of the power update equation. More recently, a utility-based power control algorithm for CDMA systems operating over flat-fading channels with matched-filter receivers was developed that takes delay, throughput, bit-error-rate, and other requirements into account implicitly through the use of “natural” utility functions (sigmoids) that can be adapted by each user according to his service needs \cite{82}. This development is particularly pleasing since it allows the system to efficiently serve virtually any kind of traffic requirement, from voice to video messaging, simultaneously. Another contribution of the present work is a modification of this algorithm for use with multiuser detectors that must be estimated from the received signals because of limited information available at the receiver. These receivers are termed
blind or group-blind as appropriate. Blind detectors are constructed with information about the channel and spreading code of only one user and are generally appropriate for downlink transmissions [10, 83]. Group-blind detectors are constructed with knowledge of more than one (but less than all) of the channels and spreading codes in the system and are usually appropriate for uplink transmissions [3].

In summary, this work considers adaptive optimization of spreading codes and transmitter powers both separately and jointly to maximize utility with the lowest possible transmit power in a heterogeneous traffic environment where adaptive blind or group-blind linear multiuser detection is employed. We consider systems that are able to adapt to changes in the channel, traffic, number of users, etc., in practical channel environments. In light of the recent work in [12], we will consider the effects of both multiple-access interference and estimation error on our receiver. The contributions of this work include:

1. A blind or group-blind adaptive algorithm for adapting spreading codes to maximize SINR in fading multipath environments.

2. A utility-based power control algorithm for CDMA systems using adaptive blind or group-blind multiuser detectors.

3. A practical receiver design including joint adaptive power control and spreading code optimization that improves the performance of adaptive CDMA systems servicing heterogeneous traffic in dispersive channel environments.

The remainder of this chapter is organized as follows. Section 2 contains a problem statement and motivates our solution. Section 3 contains a description of the system model and some background material. Section 4 presents adaptive spreading code optimization and discusses its performance. Section 5 develops a utility-based power control for CDMA with blind or group-blind multiuser detection. Section 6
presents a multiuser transmitter/receiver that employs joint spreading code optimization and utility-based power control. Finally, Section 7 contains concluding remarks.

A. Motivation and Problem Statement

We define the achievable performance region of a $K$-user CDMA multiuser receiver as the set of attainable SINR $K$-vectors given a common maximum transmitted power constraint. This definition can be used in the context of uplink or downlink transmission. In general these regions depend upon the detector and how it is computed, the spreading sequences, the maximum power constraint, the channel, relative delays for asynchronous systems, and the distance between the users and the base station.

Consider, for example, a 2-user discrete-time synchronous CDMA system operating over an AWGN channel. Assuming a fourth-order propagation law, the 2-dimensional sufficient statistics (i.e., matched filter outputs) can be written as

$$
\begin{bmatrix}
y_1[i] \\
y_2[i]
\end{bmatrix} =
\begin{bmatrix}
1 & \rho \\
\rho & 1
\end{bmatrix}
\begin{bmatrix}
\frac{\sqrt{p_1}}{d_1} & 0 \\
0 & \frac{\sqrt{p_2}}{d_2}
\end{bmatrix}
\begin{bmatrix}
b_1[i] \\
b_2[i]
\end{bmatrix}
+ 
\begin{bmatrix}
n_1[i] \\
n_2[i]
\end{bmatrix},
$$

where $\rho = c_1^T c_2$ is the cross correlation between the normalized (discrete) spreading codes, $c_1$ and $c_2$, of the two users, $p_1$ and $p_2$ are the transmit powers of Users 1 and 2 respectively, $d_1$ and $d_2$ are the distances between the base station and the mobile units of Users 1 and 2 respectively, $b_1[i], b_2[i] \in \{+1, -1\}$ are the data bits of the two users, and $n[i] \sim \mathcal{N}(0, \sigma^2 R)$. When a linear MMSE multiuser detector is employed, the SINR for User 1 is given by

$$
\text{SINR}_1 = \left[ \frac{\sqrt{p_1}}{d_1^2} + \frac{\rho \sqrt{p_2}}{d_1^2} + \sigma^2 \right]^{-1} \left[ \frac{\rho \sqrt{p_2}}{d_1^2} + \frac{p_2}{d_2^2} + \sigma^2 \right].
$$

(6.2)
Figure 17 contains the performance regions for this scenario for spreading code correlation values of $\rho = 0.7, \rho = 0.3,$ and $\rho = 0$ where $\sigma^2 = 1, d_1 = 1, d_2 = 1/0.9,$ and the maximum power for each user is 20.

The goal of power control is, essentially, to determine the “best” operating point within the achievable performance region. Four specific operating points are highlighted in Figure 17 as A, B, C and D for $\rho = 0.7$. Early work in power control consisted, essentially, of SINR balancing [80], i.e., the operating point for each user is simply the maximum commonly achievable SINR value. This corresponds to point A in Figure 17. Another option is simply to operate at maximum power. This corresponds to operating point B.

One of the most interesting approaches to choosing the operating points is utility maximization [81, 85, 86, 87], where the operating points for each user are chosen in a semi-cooperative fashion by maximizing an appropriate objective function that quantifies the benefit and cost of each point in the achievable performance region. In [81], for example, the authors defined an appropriate utility function for data as

$$\mathcal{Y}_k = \frac{LR(1 - \text{BER}_k)^M}{Mp_k} \text{ bits/Joule} \quad (6.3)$$

where $L$ is the number of information bits per packet, $M$ is the packet size ($M \geq L$), $R$ is the rate in bits per second, $p_k$ is the transmitted power of User $k$, and BER$_k$ is the bit-error-rate of User $k$. This utility function explicitly takes into account throughput and delay and can be interpreted as the number of information bits received per Joule of energy expended. Figure 18 contains a plot of the achievable utility surface for the SINR region of Figure 17 where $M = 80, R = 10000, L = 8000,$ and $\text{BER}_k = Q(\sqrt{\text{SINR}_k})$ [21]. The operating points in Figure 17 have also been annotated with the corresponding sum of the two users’ utilities, as defined by (6.3). If we choose to operate at the point that maximizes the sum of the users’ utilities, we would
Fig. 17. Achievable performance regions for two-user CDMA uplink for spreading sequence correlation values of $\rho = 0.7$, $\rho = 0.3$, and $\rho = 0$. The distances from the base station are $d_1 = 1$, $d_2 = 1/0.9$, and the total power of each user is limited to 20. The SINR is calculated via (6.2).
operate at point C. Notice, however, that point C requires that User 2 turn off his transmitter. This illustrates the basic fairness problem that often accompanies power control. The problem is that unconstrained maximization of utility, throughput, etc. often means that users that are more distant from the base or that face poor channel conditions must turn off their transmitters. This violates a basic sense of fairness that says that we should find a balance between global system performance and individual user performance. Hence, researchers have introduced fairness constraints on the maximization problem. As a simple example, we may insist that $\gamma_1/\gamma_2 \in [1/5, 5]$, i.e., neither user has more than five times the utility of the other user. This corresponds to operating point D. With this fairness constraint in place our total system performance (measured by the sum of the users’ utilities) is reduced by almost two-thirds relative to point C. Although this approach is interesting, we lack a practical method of implementing a power control algorithm using utility as defined in (6.3). Instead, we will extend an existing alternative approach to CDMA with blind and group-blind multiuser detection in the following sections.

It is also immediately evident from Figure 17 that the presence of correlation dramatically constricts the performance region. Since adaptation of spreading codes is largely ignored in the literature, one of the goals of this work is an adaptive algorithm for updating spreading codes using information about the current channel conditions.

The example we have seen corresponds to an idealized case in which the spreading sequences and received powers of all users are known. In practice, however, at least some (all but one in the blind case) of the spreading sequences are unknown. Hence the detectors, blind or group-blind as appropriate, must be estimated from the received data via a sample autocorrelation matrix or some other statistic [3, 11, 10, 88, 89, 90, 91]. With this in mind, several authors investigated the BER and SINR performance of blind and group-blind linear multiuser detectors
Fig. 18. Achievable utility surface corresponding to the SINR region of Figure 17. The utility is calculated using (6.3) with parameters $M = 80$, $R = 10000$, $L = 8000$, and $\text{BER}_k = Q(\sqrt{\text{SINR}_k})$. 
[12, 13, 14, 15, 16, 17]. One of the interesting results of this work is that increasing
transmitter power beyond a certain threshold results in diminishing returns on the
achieved SINR. In fact, the SINR converges to a constant less than infinity (see Figure
25 on page 137). The “saturation effect” suffered by blind and group blind detectors
is in contrast to the fact that the achievable SINR of exact MMSE detectors increases
without bound as transmit power increases. More specifically, the output SINR for
User 1 in a 2-user CDMA system using a subspace blind MMSE multiuser detector
over an AWGN channel, taking into account detector estimation error, is given by
[12]
\[
\text{SINR}_1 = \frac{p_1 d_1^{-4} \lambda_{12}^2}{p_2 d_2^{-4} \lambda_{12}^2 + \sigma^2 \xi + \frac{1}{M} \left[ 3 \lambda_{11} - 2 \sum_{k=1}^{2} p_k^2 d_k^{-8} \lambda_{1k}^2 \lambda_{kk} + (N - 2) \varphi \right]}
\]  
(6.4)
where
\[
\lambda_{l,k} \triangleq \frac{d_l^4}{p_l} \left[ R (R + \sigma^2 P^{-1})^{-1} \right]_{k,l}^{-1}, \quad k, l = 1, 2,
\]  
(6.5)
\[
\xi \triangleq \frac{d_2^4}{p_2^2} \left[ (R + \sigma^2 P^{-1})^{-1} R (R + \sigma^2 P^{-1})^{-1} \right]_{1,1},
\]  
(6.6)
\[
\varphi \triangleq \frac{d_2^4 \sigma^4}{p_2^2} \left[ (R + \sigma^2 P^{-1})^{-1} P^{-1} R^{-1} \right]_{1,1},
\]  
(6.7)
with
\[
P \triangleq \text{diag} \left( \frac{p_1}{d_1^2}, \frac{p_2}{d_2^2}, \ldots, \frac{p_K}{d_K^2} \right)
\]  
(6.8)
and where \( N \) is the processing gain, \( R \) is defined in (6.1), and \( M \) is the number of
sample vectors used to estimate the detector. Figure 19 illustrates the achievable
performance regions for this system taking into account estimation error (for the
subspace blind case), multiple-access interference, and limited transmitter power.
The spreading sequences are chosen so that \( \rho = 5/7 \). The AWGN power is \( \sigma^2 = 1 \)
and \( M = 50 \). The maximum power for each user is 20. As with Figure 17, it is clear
that spreading code correlations have a significant effect on achievable performance.

Fig. 19. SINR regions illustrating the performance limiting factors for CDMA using multiuser detection. The simulation parameters are the same as for Figure 6.2 with $\rho = 0.7$.

B. System Model

In this section we present the system model including a description of the general discrete-time signal and channel models that will be used throughout the rest of the chapter and a brief review of blind and group-blind multiuser detection. Since most of this material has previously appeared in the literature, we will summarize and cite references for brevity.
1. Discrete-Time Signal Model

The following model is general in that it takes asynchronism and multipath fading into account. We consider a $K$-user sliding-window, discrete-time linear model of the form \[ r[i] = HAb[i] + v[i] \] (6.9)

where the bits that we wish to demodulate from (6.9) are $[b[i]]_{K(i+1)}$ through $[b[i]]_{K(i+1)}$, and where $\iota$ denotes the maximum total delay (path delay plus transmit delay) in symbol intervals. These bits are henceforth denoted by $\{b_k[i]\}_{k=1}$. The smoothing factor, necessary for blind channel identification, is given by $m$. If we define $r = K \cdot (m + \iota)$, then the sizes of $r[i]$, $v[i]$, $H$, $b[i]$, and $A$ are given by $Nm \times 1$, $Nm \times 1$, $Nm \times r$, $r \times 1$, and $r \times r$ respectively, where $N$ is the system processing gain. Note that $A$ contains the users’ transmit powers and is a block diagonal matrix of the form

\[
A = \begin{bmatrix}
P & 0 \\
& \ddots \\
0 & P \end{bmatrix}_{r \times r} \tag{6.10}
\]

where $P = \text{diag}(\sqrt{p_1}, \sqrt{p_2}, \ldots, \sqrt{p_K})$. Denote by $D$ the matrix with the same structure as $A$ with $\sqrt{p_1}, \sqrt{p_2}, \ldots, \sqrt{p_K}$ replaced with the corresponding distances $d_1, d_2, \ldots, d_K$.

The columns of $H$ that correspond to the bits in $\{b_k[i]\}_{k=1}^K$ are the composite signature waveforms and are given by $h_k \triangleq He_{K\iota+k}$ for $k = 1, \ldots, K$ where $e_{K\iota+k}$ is the vector whose entries are all zero except the $(K\iota+k)$-th entry, which is one. We assume that the complex path gains for each user are normalized such that the composite signature waveforms satisfy

\[
\|h_k\|^2 = \frac{C_0}{d_k^2} \tag{6.11}
\]
for $k = 1, \ldots, K$ where $d_k$ is the distance from the base station to the mobile of User $k$ and $C_0$ is a constant that depends on antenna gains, signal wavelengths, etc. For convenience, $C_0$ is set such that a single user at 1000 meters from the base station transmitting at 10 Watts over a non-fading AWGN channel will achieve an SNR of 15dB. Note that the columns of $H$ (the composite signature waveforms) contain information about both the timings and the complex path gains of the multipath channel of each user. Hence an estimate of these waveforms eliminates the need for separate estimates of the timing information.

2. Discrete-Time Channel Model

The continuous-time channel model for User $k$ that is implicit in (6.9) is given by

$$g_k(t) = \sum_{l=1}^{L} \alpha_{kl} \delta(t - \tau_{kl})$$

(6.12)

where $\alpha_{kl}$ is the complex path gain associated with the $l$-th path for the $k$-th user, and $\tau_{kl}, \tau_{k1} < \tau_{k2} < \cdots < \tau_{kL}$ is the sum of the associated path and initial transmission delays of User $k$. Define the sequence $f[\cdot]$ as

$$f[n] = \sum_{l=1}^{L} \alpha_{kl} \int_{0}^{T_c} \psi(t) \psi(t - \tau_{kl} + nT_c) dt$$

(6.13)

where $T_c$ is the chip interval and $\psi(t)$ is a normalized chip waveform of duration $T_c$. We can see that $f[n]$ is zero whenever $n < 0$ or $n > \tau N$. With this in mind we denote the discrete-time channel response for User $k$ by

$$f_k \triangleq [f_k[0] \cdots f_k[\tau N]]^T.$$  

(6.14)
If we also define

$$\overline{C}_k \triangleq \begin{bmatrix} c_k & 0 & \cdots & 0 \\ 0 & c_k \end{bmatrix}^{N(I+1) \times (N(I+1))}$$

and

$$\overline{F}_k \triangleq \begin{bmatrix} f_k & 0 & \cdots & 0 \\ 0 & f_k \end{bmatrix}^{N(I+1) \times N}$$

where $c_k \triangleq \left[ c_k[1] \ c_k[2] \cdots c_k[N] \right]^T$ is the normalized spreading code of User $k$, then we may write the composite signature waveforms in (6.11) as [3, 11]

$$h_k = \overline{C}_k f_k = \overline{F}_k c_k$$

for $k = 1, 2, \ldots, K$.

3. A Review of Blind and Group-Blind Multiuser Detection

Since the focus of this chapter is on transmitter optimization for blind and group-blind multiuser detection, we briefly review these detectors in this section. Note that $E\{\cdot\}$ will denote ensemble averaging. Since the ambient noise is white, i.e., $E\{v[i]v[i]^H\} = \sigma^2 I_{Nm}$ where $I_{Nm}$ is the $Nm \times Nm$ identity matrix, and since the transmitted bits are assumed uncorrelated, the autocorrelation matrix of the received signal in (6.9) is

$$C_r \triangleq E\{r[i]r[i]^H\} = H A^2 H^H + \sigma^2 I_{Nm}$$

where

$$\begin{align*}
C_r &= \begin{bmatrix} U_s \Lambda_s U_s^H + \sigma^2 U_n U_n^H, \\
(6.18)
\end{align*}$$

where $\frac{6.18}{6.18}$ is the eigendecomposition of $C_r$. $U_s$ has size $Nm \times r$ and $U_n$ has size $Nm \times (Nm - r)$.

The MMSE multiuser detector and corresponding bit estimate for $b_k[i]$ are given
The solution to (6.19) can be written in terms of the signal subspace components as

\[ w_k[i] = U_s \Lambda_s^{-1} U_s^H h_k. \]  

(6.21)

This detector can be implemented in a blind fashion, where the receiver has knowledge only of the signature waveform of the user of interest, by estimating the signal subspace components, \( U_s, \Lambda_s \), from the received signal. This can be accomplished using the sample autocorrelation matrix of the received signal or via subspace tracking [63]. We also need to use some form of blind channel estimation, which will be discussed in Section 1.

There are some situations in which the receiver may have knowledge of \( \tilde{K}, 1 < \tilde{K} < K \) signature waveforms, e.g., uplink CDMA when inter-cell interference is present. With this additional information, we can develop detectors that outperform the blind implementations of (6.21). A set of these “group-blind” detectors was developed in [3]. Define the set of matrices \( \{ \tilde{H}_j \}_{j=0}^{m+i-1} \) such that \( \tilde{H}_j \) is the \( Nm \times \tilde{K} \) matrix composed of columns \( jK + 1 \) through \( jK + \tilde{K} \) of the matrix \( H \). We define the matrix \( \tilde{H} \overset{\Delta}{=} [\tilde{H}_0 \quad \tilde{H}_1 \cdots \tilde{H}_{m+i-1}]. \) The size of \( \tilde{H} \) is \( Nm \times \tilde{r} \) where \( \tilde{r} \overset{\Delta}{=} \tilde{K}(m+i). \) Define the matrix \( \tilde{A} \) similarly. Then the group-blind linear hybrid detector for User \( k \quad (1 \leq k \leq \tilde{K}) \) is given by the solution to the following constrained optimization problem:

\[ w_k = \arg \min_{w \in \text{range}(H)} E \left\{ |b_k[i] - w^H r[i]|^2 \right\} \]  

(6.22)
subject to the constraint $\mathbf{w}^H \hat{\mathbf{H}} \hat{\mathbf{A}} = e^T_{K+i+k}$. Heuristically speaking, this detector zero-forces the interference caused by the $\tilde{K}$ known users, and suppresses the interference from unknown users according to the MMSE criterion. The solution and the corresponding bit estimate for User $k$ may be written as [3]

$$
\mathbf{w}_k = U_s \Lambda_s^{-1} U_s^H \hat{\mathbf{H}} \hat{\mathbf{A}} \left[ \hat{\mathbf{A}}^T \hat{\mathbf{H}}^H U_s \Lambda_s^{-1} U_s^H \hat{\mathbf{H}} \hat{\mathbf{A}} \right]^{-1} e_{K+i+k} \quad (6.23)
$$

$$
\hat{b}_k[i] = \text{sign} \left[ \text{Re} \{ \mathbf{w}_k^H r[i] \} \right], \quad k = 1, 2, \ldots, \tilde{K}. \quad (6.24)
$$

C. Blind Adaptive Spreading Code Optimization

In this section, we propose a method of optimizing spreading codes that takes into account channel conditions and multiple-access interference. This algorithm can be implemented in blind or group-blind fashion and, therefore, is appropriate for both uplink and downlink transmissions. Choosing optimal sequences for synchronous CDMA over a non-fading AWGN channel when $K \leq N$ is a trivial problem: use orthogonal sequences. The problem has been investigated in [76] for situations in which $K > N$. Therefore, we restrict our attention to adapting codes for the fading multipath channel model discussed in Section 1. This problem is relevant since spreading code sets with good correlation properties can, after convolution with the channel, lead to composite signature waveform sets with poor correlation properties.

1. Maximum Eigenvector Method

There are a number of optimization problems that we can formulate with the stated goal of improving performance. We may, for example, form optimal codes by mini-
mizing composite signature waveform correlations via

\[
\arg \min_{c_1, \ldots, c_K} \left\| [h_1 \cdots h_N]^H [h_1 \cdots h_N] \right\|_1 = \arg \min_{c_1, \ldots, c_K} \sum_{j=1}^{K} \sum_{k=1}^{K} |c_j^T \overline{F}_j \overline{F}_k c_k| \tag{6.25}
\]

where \( \| \cdot \|_1 \) is the \( l_1 \) matrix norm defined as the sum of the absolute value of each of the matrix elements. In this chapter we choose a different, more direct, approach by noting from (6.9) that the SINR for User \( k \) \( (1 \leq k \leq K) \), when \( H \) is perfectly known and an MMSE multiuser detector is employed, is given by [84]

\[
\Xi_k \tag{6.26}
\]

\[
\text{SINR}_k = \frac{C_0 p_k d_k^2}{d_k} \frac{\overline{F}_k H}{T_k} \frac{C_0 [A]^2}{[D]^2} \sum_{j=1}^{K} \frac{C_0 [A]_{jj}^2 H_j H_j^H + \sigma^2 I_{Nm}}{[D]_{jj}^2} \frac{\overline{F}_k c_k}{ \Xi_k c} \tag{6.26}
\]

where \( H_j \) denotes the \( j \)-th column of \( H \) and \( [A]_{jj} \) denotes the element in the \( j \)-th column and \( j \)-th row of the matrix \( A \). Equation (6.26) suggests a strategy in which each user independently chooses his new spreading sequence, \( c_k^{\text{new}} \), to satisfy

\[
c_k^{\text{new}} = \arg \max_{c \in \{-1, +1\}^N} c^T \overline{\Xi}_k c. \tag{6.27}
\]

The solution to a related problem,

\[
\max_{c \in \mathbb{C}^N} c^H \overline{\Xi}_k c, \quad \|c\| = 1 \tag{6.28}
\]

can be found, when \( \overline{\Xi}_k \) is independent of \( c \), using the eigenvector corresponding to the maximum eigenvalue of \( \overline{\Xi}_k \) [92, pp. 176-177]. Henceforth, we will refer to this eigenvector as the maximum eigenvector of \( \overline{\Xi}_k \). Then we may set \( \sqrt{N} c_k^{\text{new}} \) equal to the sign of the real part of the maximum eigenvector of \( \overline{\Xi}_k \). Strictly speaking, however, \( \overline{\Xi}_k \) is not always independent of \( c_k \), the spreading code of the \( k \)-th user. Whenever \( m + \nu > 0 \) there is some weak dependence in that 1 out of every \( K \) columns
of $H$ has some dependence on $c_k$. Despite this weak dependence, this algorithm provides a substantial increases in achievable SINR.

To develop a blind or group-blind implementation of this algorithm we need blind estimators of $\{\xi_k\}_{k=1}^{\tilde{K}}$ and of the channels of each known user, $\{f_k\}_{k=1}^{\tilde{K}}$. Note that

\[
\xi_k = \overline{F}_k^H \left[ H A^2 H^H + \sigma^2 I_{Nm} - \frac{C_0 p_k}{d_k^4} h_k h_k^H \right]^{-1} \overline{F}_k
\]

\[
= \overline{F}_k^H \left[ C_r - \frac{C_0 p_k}{d_k^4} h_k h_k^H \right]^{-1} \overline{F}_k
\]

\[
= \overline{F}_k^H \left[ C_r^{-1} + \frac{C_0 p_k/d_k^4}{1 - C_0 p_k/d_k^4} C_r^{-1} h_k h_k^H C_r^{-1} h_k h_k^H C_r^{-1} \right] \overline{F}_k. \tag{6.29}
\]

We can estimate $C_r^{-1}$ via (blind) batch eigendecomposition or subspace tracking of the received signal since

\[
C_r^{-1} = [U_s \Lambda_s U_s^H + \sigma^2 U_n U_n^H]^{-1}
\]

\[
= \left[ \begin{array}{cc} U_s & U_n \end{array} \right] \left[ \begin{array}{cc} \Lambda_s^{-1} & 0 \\ 0 & \frac{1}{\sigma^2} I_{Nm-r} \end{array} \right] \left[ \begin{array}{c} U_s^H \\ U_n^H \end{array} \right]. \tag{6.30}
\]

In order to track the channels of the $\tilde{K}$ known users, we can use the standard blind approach of taking advantage of the orthogonality between the signal and noise subspaces. For User 1, for example, we have [93]

\[
\hat{f}_1 = \arg \min_{\|f\|=1} \|U_n^H C_1 f\|^2 \tag{6.31}
\]

\[
= \arg \min_{\|f\|=1} f^H \left( C_1^H U_n U_n^H C_1 \right) f \tag{6.32}
\]

\[
= \text{minimum eigenvector of } Q \tag{6.33}
\]

where the noise subspace, $U_n$, can be estimated from subspace tracking or batch eigendecomposition of the received signal. Alternatively, we can use the blind sequential adaptive Kalman channel estimator in [94]. The application of this algo-
rithm to the present context is straightforward and will not be discussed in detail. These blind channel estimators incur channel phase and amplitude ambiguities. The phase ambiguities can be overcome using a phase estimator, as in [13, Eq. 124], or by differential encoding/decoding of the data. Since $C_0$ is assumed known, as is $p_k, d_k$ for each known user, the channel amplitudes can be determined using estimates of the received signal power.

**Algorithm 4.** [Blind Adaptive Spreading Code Optimization via the Maximum Eigenvector Method]

1. Obtain estimates of the signal subspace parameters $U_s, U_n, \Lambda_s$, and $\sigma^2$ via subspace tracking [63] or batch eigendecomposition of the received signal.

2. Obtain blind estimates of the channel, $f_k$, for the $\bar{K}$ ($\bar{K} \leq K$) known users. For the blind scenario (downlink), $\bar{K} = 1$; for the group-blind scenario (uplink), $1 < \bar{K} \leq K$.

3. If $\bar{K} = 1$, set $Q = 1$, otherwise set $Q$ equal to the desired number of iterations and iterate the following procedure $Q$ times.

   For $k = 1, 2, \ldots, \bar{K}$

   (a) For User $k$, calculate $\Xi_k$ using (6.29) and set $u_1$ equal to the maximum eigenvector of $\Xi_k$.

   (b) Set the new binary bipolar spreading code for User $k$ to be

   $${c_k^{\text{new}} = \text{sign} \{\text{Re} \{u_1\}\}.} \quad (6.34)$$

   (c) Update $C_r^{-1}$ iteratively by removing the influence of the old spreading code information:
For $j = 0, 1, \ldots, m + \iota - 1$

$$C_r^{-1} \leftarrow C_r^{-1} + \frac{C_0 p_k / d_k^4}{1 - C_0 p_k / d_k^4 H_{K_j+k}^H C_r^{-1} H_{K_j+k} C_r^{-1}} C_r^{-1} H_{K_j+k} H_{K_j+k}^H C_r^{-1}.$$ \hspace{0.5cm} (6.35)

(d) Using $F_k$ and $c_k^{\text{new}}$, update the columns of $H$, $\{H_{K_j+k}\}_{j=0}^{m+\iota-1}$, that are affected by the new code.

(e) Update $C_r^{-1}$ iteratively with the new spreading code information:

For $j = 0, 1, \ldots, m + \iota - 1$

$$C_r^{-1} \leftarrow C_r^{-1} - \frac{C_0 p_k / d_k^4}{1 + C_0 p_k / d_k^4 H_{K_j+k}^H C_r^{-1} H_{K_j+k} C_r^{-1}} C_r^{-1} H_{K_j+k} H_{K_j+k}^H C_r^{-1}. \hspace{0.5cm} (6.36)$$

Complexity: Note that subspace tracking and channel estimation are necessary for the detection process. Furthermore, the maximum eigenvector computation in step 3a can be computed with $O(N^2)$ floating point operations per user per iteration using, for example, the power method [95]. Therefore, the additional computational complexity incurred by spreading code optimization per user per iteration is dominated by the $6(m + \iota)$ vector outer products and matrix-vector products in steps 3c and 3e, each of which has complexity $O(N^2)$. Assuming the channel is relatively constant over a block of data, we need only perform code optimization once per block. If the block length is $\bar{M}$, then the computational complexity per user per iteration per symbol is then $O \left( (m + \iota) N^2 / \bar{M} \right)$. The total complexity per symbol is then $O \left( Q \bar{K} [m + \iota] N^2 / \bar{M} \right)$. Note that $m$ and $\iota$ are generally $O(1)$ and $\bar{M}$ can be $O(10^3)$ for high data rate systems.

2. Extensions to Non-binary Codes

It is natural to expect that we should be able to improve upon Algorithm 1 by taking advantage of the degrees of freedom that are eliminated by the sign$\{\cdot\}$ and Re$\{\cdot\}$ functions used to obtain $c_k^{\text{new}}$. The use of QPSK modulation instead of BPSK,
for example, results in baseband complex spreading codes of the form $\sqrt{2N}c_k \in \{1 + j, 1 - j, -1 + j, -1 - j\}^N$. If we also replace the typical binary shift-adder sequence generator with a layer of chip-level modulation, we can generate complex codes that vary (almost) continuously, that is, $c_k \in \mathbb{C}^N, \|c_k\| = 1$. In light of the (baseband) complex and continuously varying channel model, we would expect that these additional degrees of freedom would enable us to generate superior composite signature waveform sets. We will evidence of this next.

3. Simulation Results

Here we will quantify the SINR gain and adaptation ability of Algorithm 1 and its non-binary extension. We assume perfect power control for the simulation results in this section in that $\|\sqrt{p_1}h_1\| = \|\sqrt{p_2}h_2\| = \cdots = \|\sqrt{p_K}h_K\| = 1$, though these conditions are not necessary for Algorithm 1. In Section E, we will see the performance of spreading code optimization in a receiver employing a practical power control algorithm.

Figure 20 contains plots of the SINR for the $\tilde{K} = 4$ known users, calculated via (6.26), in a fading multipath environment. The pre-detection SNR, defined for user $k$ as $\|h_k\|^2/\sigma^2$, is set to 9 dB for each initial random code set. Since $\|h_k\|^2$ depends upon $c_k$, the spreading code for user $k$, the pre-detection SNR after spreading code optimization may be slightly larger. This can be avoided by re-normalization after each iteration, but this would be a misleading approach since the physical interpretation would be that the channel depends upon the spreading code. The processing gain is 15 and the total number of users is 5. The chip pulse is a raised cosine with roll-off factor .5. Each user has $L = 3$ paths and the delay of each path is uniformly distributed over a single symbol period. Hence, the maximum delay spread is one symbol interval, i.e., $\tau = 1$. The fading gains for each user’s channel are generated
from a complex Gaussian distribution and the smoothing factor is $m = 2$. The algorithm performance results in this figure represent an average over 50 random initial code sets for each of 400 random channels. Each of the 400 random channels has been estimated using the batch blind channel estimator in (6.33), where the noise subspace has been estimated using a batch eigendecomposition of 600 received signal samples. The phase ambiguities are resolved in these simulations using the simple fourth-order phase estimator in [13]. Note that this phase estimator still contains a phase ambiguity of $\pi$ for BPSK, which is inherent to any blind estimator. The channel amplitude ambiguities have been circumvented here by assuming perfect power control. The first bar for each user represents the average performance without any spreading code optimization. Since these results represent an average over random initial code sets and random channels, the bars are nearly identical for each user. The second bar represents the performance after 1 iteration of Algorithm 1 using binary codes. The third and fourth bars represent, respectively, the performance after 3 and 5 iterations. Notice that after 5 iterations, we see a 35% average improvement in SINR using binary codes.

Figure 21 has been generated in a manner identical to Figure 20, except that the optimized spreading codes have been allowed to be complex and continuously varying. We now see an 83% average improvement in SINR after 5 iterations relative to a non-optimized system. This figure also makes it clear that the SINR improvement will not necessarily be evenly distributed among all users after 1 iteration. Notice, in particular, that User 1’s performance after 3 iterations is worse, on average, than his performance after 1 iteration. This is because the system-wide SINR improvement is more evenly distributed after the third iteration.

In Figure 22, the number of known users has been reduced to 1, i.e., this is the blind case. Iteration is not helpful since there is only one user to optimize, so
the results are for 1 iteration. As before, the first bar is the performance without any spreading code optimization. The second bar is for optimization with binary codes, and the third bar is for optimization using complex and continuously varying spreading codes.

Figure 23 represents the performance of group-blind code optimization for a particular fixed channel, averaged over 300 random initial code sets. Notice that we see in this figure a problem that is often reported in the power control literature in that performance varies greatly among the users in the system, even after a large number of iterations. This is a fairness problem induced by poor channel conditions that can be addressed in the power control layer. However, the coupling of the power control and spreading code optimization problems suggests the use of joint power control and code optimization which will be discussed in Section 1.


In this section we develop a utility-based power control algorithm for CDMA systems using blind and group-blind multiuser detectors. Since these detectors must be estimated from the received signal, their performance characteristics differ significantly from those of perfectly-known detectors, e.g., MMSE, decorrelator. As we will see, these characteristics must be taken in to account to develop a power control algorithm that does not waste system resources. Although we will discuss in detail the group-blind (uplink) scenario, we will also indicate the non-trivial modifications that are necessary for the blind (downlink) case.

In general, a good power control algorithm should have the following properties:

- It should be distributed, simple, agile, robust, and scalable [96].
Fig. 20. Average SINR for the 4 known users before and after group-blind spreading code optimization using binary codes. Results are shown for no optimization, 1 iteration, 3 iterations, and 5 iterations. The SNR is fixed at 9dB for each initial code set.
Fig. 21. Average SINR for the 4 known users before and after group-blind spreading code optimization using complex and continuously varying codes. Results are shown for no optimization, 1 iteration, 3 iterations, and 5 iterations. The SNR is fixed at 9dB for each initial code set.
Fig. 22. Average SINR for the known user before and after blind spreading code optimization. Results are shown for no optimization, optimization with binary codes, and optimization with complex and continuously varying codes. The SNR is fixed at 9dB for each initial code set.
Fig. 23. Average SINR before and after group-blind spreading code optimization for one particular channel realization. Results are shown for no optimization, 1 iteration, 3 iterations, and 5 iterations. The SNR is fixed at 9dB for each initial code set.
- It should have the capacity to serve heterogenous traffic with varying performance requirements.

- It should satisfy as many properties of the Yates standard power control framework [97] as possible, e.g., convergence for asynchronous case, high Pareto efficiency ¹.

- It should provide a method of assuring fairness for users who are distant from the base station or who face poor channel conditions.

- It should gracefully adapt when dynamic conditions render the quality-of-service (QoS) requirements infeasible.

Utility-based power control algorithms are attractive in that they acknowledge that transmitter power is a valuable resource. Users are not willing to achieve QoS requirements at arbitrarily high transmitter power levels. Utility-based algorithms take this into account by formulating the problem using microeconomics and game-theoretic concepts. Typically, each user chooses his transmitter power independently by maximizing a net utility function, given by $U_k(\text{SINR}_k) - C_k(p_k)$ where $U_k(\cdot)$ is an appropriately defined utility function and $C_k(\cdot)$ is a kind of cost function that penalizes high transmit power and leads to a more Pareto efficient result than can be obtained by maximizing $U_k(\text{SINR}_k)$ alone. The resulting algorithm is a $K$-person non-cooperative game. Unfortunately, existing works in utility-based power control have significant limitations. The approach in [81], for example, leads to a high-complexity power updating algorithm. Moreover, the utility function in [81] is designed for data and is not

¹A power assignment vector is Pareto optimal if it is the componentwise minimum vector among the set of all power assignments that satisfy the performance requirements. A Pareto efficient algorithm tends to produce Pareto optimal power assignments.
be appropriate for other types of traffic. In [82], the authors designed an alternative utility-based algorithm that can be tuned to satisfy all the requirements listed at the beginning of this section and that avoids the problems associated with the approach in [81]. However, this work is not applicable to CDMA with multiuser detection. In this section we will use the algorithm in [82] as a basis for developing a new power control algorithm that is appropriate for use with blind and group-blind multiuser detectors. We begin with a review of the basic algorithm.

1. Basic Algorithm

Instead of setting his SINR constraint directly, as is required by most conventional power control algorithms, User $k \ (1 \leq k \leq K)$ defines a utility function, $U_k(\cdot)$, that is appropriate for his traffic type. The user then sets his transmitter power to the value that maximizes the corresponding net utility given by

$$\Gamma_k(\text{SINR}_k, p_k) \triangleq U_k(\text{SINR}_k) - C_k(p_k)$$

(6.37)

where $C_k(p_k)$ is a penalty function, usually of the form $C_k(p_k) = \zeta_k p_k$ where $\zeta_k$ is a price coefficient for User $k$. The utility functions are chosen from the family of sigmoid functions that are parameterized by their center point, $\beta_k$, and their maximum slope, $\gamma_k$. Mathematically, the functions are given by

$$U_k(\text{SINR}_k) = \left(1 + e^{-4\gamma_k(\text{SINR}_k - \beta_k)}\right)^{-1}$$

(6.38)

and are illustrated in Figure 24. We will discuss how utility functions and price coefficients are chosen in a later section.

Mathematically, the transmitter power for User $k$ is chosen according to

$$p_k^{\text{set}} = \arg \max_{p \geq 0} \Gamma_k.$$  

(6.39)
If there exists a positive $p$ that maximizes (6.39), the solution may be found via

$$\frac{\partial (\Gamma_k(\text{SINR}_k, p_k))}{\partial p_k} \bigg|_{\text{set}} = 0 \Rightarrow U'_k(\text{SINR}_k) = \frac{\zeta_k}{\partial (\text{SINR}_k)/\partial p_k}. \tag{6.40}$$

The work in [82] assumes a single-path ($L = 1$), real-valued, flat-fading channel with a post detection SINR model of the form

$$\text{SINR}_k = \frac{\alpha_k^2 C_0 p_k / d_k^4}{\sum_{j \neq k} \alpha_j^2 C_0 p_j / d_j^4 + \sigma^2}. \tag{6.41}$$

This is applicable to TDMA/FDMA or synchronous CDMA with matched-filter receivers. If we employ a perfectly-known MMSE detector, the post-detection SINR for synchronous CDMA operating over a similar channel is of the form

$$\text{SINR}_k = \frac{C_0 p_k \alpha_k^2}{d_k^4} c_k^T \left[ \sum_{j \neq k} \frac{C_0 p_j \alpha_j^2}{d_j^4} c_j c_j^T + \sigma^2 I_N \right]^{-1} c_k. \tag{6.42}$$

For both (6.41) and (6.42) we may write

$$\text{SINR}_k = Z_k p_k \ \Rightarrow \ \frac{\partial (\text{SINR}_k)}{\partial p_k} = Z_k \tag{6.43}$$

where $Z_k$ is independent of $p_k$. When blind or group-blind detectors are used, SINR$_k$ is a highly nonlinear function of $p_k$ and $Z_k$ is not independent of $p_k$. We will address the necessary modifications later. For now we assume $Z_k$ is independent of $p_k$. If we define the set of functions $\{f_k(\cdot)\}_{k=1}^K$ such that $f_k(\text{SINR}_k) = U'_k(\text{SINR}_k)$ over the concave part of $U_k$ (where a local maximum is possible) and substitute (6.43) into (6.40), we find the target SINR for User $k$ that corresponds to the solution of (6.39)

\footnotetext[2]{This set of continuous-time functions is not to be confused with the channel vectors $\{f_k\}_{k=1}^K$. They are completely unrelated, though we retain the notations to be consistent with existing literature.}
to be

$$\text{SINR}_k = f_k^{-1} \left( \frac{\zeta_k}{Z_k} \right).$$  \hfill (6.44)

Since $p_k = \text{SINR}_k/Z_k$, the corresponding power assignment is given by

$$\hat{p}_k = \frac{1}{Z_k} f_k^{-1} \left( \frac{\zeta_k}{Z_k} \right).$$  \hfill (6.45)

Since this is an iterative algorithm, we add a time index, $n$, and form the update equation,

$$\hat{p}^*_k(n + 1) = \frac{1}{Z_k(n)} f_k^{-1} \left( \frac{\zeta_k}{Z_k(n)} \right) = \frac{\text{SINR}_k(n)}{\text{SINR}_k(n)} \hat{p}_k(n)$$  \hfill (6.46)

where SINR$_k$ is the current SINR for User $k$. The optimal power at step $n$, given by $\hat{p}_k(n + 1)$, is either $\hat{p}^*_k(n + 1)$ or zero, depending on the slope of the penalty function associated with User $k$. If the net utility associated with $\hat{p}_k(n + 1) = \hat{p}^*_k(n + 1)$ is less than or equal to zero, we should set $\hat{p}_k(n + 1) = 0$. Figure 24, for example, illustrates the penalty slope threshold for $\beta_k = 10$ and $\gamma_k = 1.0$. If the slope of $C_k(p_k) = \zeta_k p_k = \zeta_k \cdot \text{SINR}_k/Z_k$, which is given by $\zeta_k/Z_k$, is equal to or larger than the slope of the indicated line, then the net utility will be non-positive and we should set our transmit power to zero. To be consistent with the terminology from previous works [82], we call the SINR associated with the cost threshold the **turnoff** SINR, and denote it by $\overline{\text{SINR}}_k$.

Notice that each user maximizes his utility independently of the other users. In this sense, utility maximization is a noncooperative game. However, cooperation emerges indirectly in that a user will decrease his target SINR when interference increases.
Fig. 24. Utility functions (sigmoids) for $\beta_k = 10$ and the turnoff cost threshold for $\gamma_k = 1.0$. 
2. Utility-Based Power Control for Blind and Group-Blind Multiuser Detectors

If a perfectly-known MMSE multiuser detector is available and the channel is single-path, flat-fading AWGN, then the utility-based power control algorithm (UBPC) in [82] can be adapted to multiuser detection for CDMA very easily, through the substitution of (6.42) for (6.41). For blind or group-blind multiuser receivers, however, the nonlinear nature of the SINR as a function of transmit power means \( Z_k \) in (6.43) is not independent of \( p_k \). We will address the necessary modifications to UBPC in this subsection. We return to the general asynchronous fading multipath CDMA signal and channel models of Section 1.

a. The Piecewise Linear SINR Approximation

Consider the SINR as a function of transmit power for the subspace blind MMSE detector when the channel is known. Define \( g_k = \sqrt{p_k} h_k \) for \( k = 1, 2, \ldots, K \) and \( \tilde{R} \triangleq [g_1 \ g_2 \cdots g_K]^H [g_1 \ g_2 \cdots g_K] \). Then the SINR for User 1 is given by [13]

\[
\text{SINR}_1 = \frac{(w_1^H g_1)^2}{\sum_{k=2}^K |w_1^H g_k|^2 + \sigma^2 \|w_1\|^2 + \frac{1}{M} \text{tr} \left( C_w^0 C_r \right)}
\]

where

\[
w_i^H g_j = \left[ I_K + \sigma^2 \tilde{R}^{-1} \right]^{-1}_{i,j}, \tag{6.48}
\]

\[
\|w_1\|^2 = \left[ \left( \tilde{R} + \sigma^2 I_K \right)^{-1} \tilde{R} \left( \tilde{R} + \sigma^2 I_K \right)^{-1} \right]_{1,1}, \tag{6.49}
\]

\[
\tau = \sigma^2 \left[ \left( \tilde{R} + \sigma^2 I_K \right)^{-1} \tilde{R}^{-1} \right]_{1,1}, \tag{6.50}
\]

\[
\text{tr} \left( C_w^0 C_r \right) = K w_1^H g_1 + (N - K) \tau \sigma^2 + \sum_{i=1}^K \sum_{j=1}^K (g_i^H w_i)(g_i^H w_j)(w_j^H g_j) - \tag{6.51}
\]

\[
2 \sum_{k=1}^K |g_k^H w_1|^2 (g_k^H w_k). \tag{6.52}
\]
Figure 25 contains a typical plot of $\text{SINR}_1$ versus $p_1$ with all other parameters held constant. For this figure, $N = 15$, $K = 9$, $\sigma^2 = 1$, $M = 550$, and the spreading codes and channel are random. Also included in this figure is the plot of $\text{SINR}_1$ versus $p_1$ when an exact MMSE detector is available. Notice that although the shape of the achievable SINR for the blind detector case is clearly non-linear, it lends itself to a simple piecewise linear approximation. One such approximation is indicated by the dotted line and is given by

$$\text{SINR}_1^a(p_1) = \begin{cases} \kappa_1 p_1 & 0 \leq p_1 \leq \eta / \kappa_1 \\ \eta & p_1 > \eta / \kappa_1 \end{cases}$$

(6.53)

where $\kappa_1$ and $\eta$ are the initial slope and asymptotic values, respectively, of the SINR approximation. This linear approximation will allow us to use UBPC with estimated detectors by replacing $Z_k$ in (6.43) with $\kappa_k$ for $0 \leq p_k \leq \eta / \kappa_k$. We will also need to modify the basic algorithm by preventing User $k$ from transmitting at a power level greater than $\eta / \kappa_k$ since power levels larger than this increase interference for other users and do not improve the SINR for User $k$.

b. Calculating the SINR Approximation Parameters

In order to maintain the group-blind (or blind, as appropriate) and distributed nature of the proposed receiver, we need to obtain estimates of $\eta$ and $\{\kappa_k\}_{k=1}^{K}$ using locally available information. A good approximation to the asymptotic SINR can be obtained via the large-system analysis of blind and group blind detectors in [14]. In that work, the authors characterized the output SINR of estimated detectors assuming binary random spreading while letting $N$, $K$, and the number of samples, $M$, go to infinity. They also assume that the ratios $\frac{K}{N}$ and $\frac{M}{N}$ are constant and that $\frac{K}{N} \leq 1$. One of
Fig. 25. Output SINR versus transmit power for User 1, along with a piecewise linear approximation for the subspace blind MMSE multiuser detector in a fading multipath channel.
their results is that the asymptotic SINR is given by

$$\eta = \frac{M}{N}.$$  \hfill (6.54)

For the parameters used in Figure 25, for example, $M/N = 550/15 \approx 36.7$, which matches the asymptotic value we see in the figure.

To compute the best fitting slope parameters directly, the base station (for the uplink) or the mobile unit (for the downlink) would require knowledge of all the spreading codes, channel states, transmit powers, and distances of all the users in the system. Since we wish to maintain the group-blind, distributed nature of the receiver, this is unacceptable. However, there is a high correlation between the best fitting initial slope and the slope of the SINR for a perfectly-known MMSE detector. That is, if we can compute the latter locally, we can estimate the former numerically based on their statistical dependence. More precisely, we can pre-compute (offline) a one-to-one mapping

$$\mathcal{X} : (0, \infty) \rightarrow (0, \infty)$$ \hfill (6.55)

that maps a locally-measurable quantity, e.g., the slope of the SINR of the exact MMSE detector, ignoring presumably distant unknown users, to $\kappa_k$, $1 \leq k \leq \hat{K}$.

One way to develop the mapping $\mathcal{X}$ is to simply to generate, via Monte Carlo simulation, ordered pairs of the form $(x, y)$ where $x$ is the SINR slope of the MMSE detector, ignoring unknown users and $y$ is an “ideal” slope found using information that is unavailable in practice. We can fit a curve to these data points, and use the curve online as the mapping since the first member of each pair can be computed locally and is given by

$$\frac{C_0}{d_k^2} c_k^T \mathbf{F}_k^H \left[ \sum_{j \in \mathcal{D}_k} \frac{C_0 \mathbf{[A]}^2_{j,j} \mathbf{H}_j \mathbf{H}_j^H}{[\mathbf{D}]_{j,j}^4} + \sigma^2 \mathbf{I}_{N_m} \right]^{-1} \mathbf{F}_k c_k$$ \hfill (6.56)
where the set \( \mathcal{D}_k \triangleq \{ Kw + i : w = 0, 1, \ldots, i + m - 1 \text{ and } i = 1, 2, \ldots, \bar{K} \text{ and } (w, i) \neq (\nu, k) \} \). Figure 26 contains a scatter plot of 500 such ordered pairs where the \( x \)-axis is the “ideal” slope and the \( y \)-axis is the corresponding SINR slope of the perfectly-known MMSE detector, ignoring any unknown users. The parameters for this simulation are \( N = 15, K = 9, \bar{K} = 7 \), and the spreading codes and channel are random and vary for each point. The ideal slope was computed by fitting (via least squares) the best line to SINR\(_1\) over the SINR range 0 to 18dB since this generally covers the SINR range of interest. The ideal slope cannot be used in practice, of course, since we require spreading code and channel information for every user in the system to evaluate SINR\(_1\) as a function of \( p_1 \). We pass the simulated data through a non-parametric local linear function estimator \([98]\) to obtain the mapping \( \mathcal{X} \) shown in the figure (solid line). We can compute and store the mapping offline since it represents an average over channel states, spreading codes, and background noise powers. Additional mapping should be stored for situations when \( N/K \) or \( \bar{K}/K \) differ significantly from the values mentioned above.

Another option that is particularly useful for situations in which \( \bar{K}/K \) is small, e.g., the blind case, is to use an estimate of the SINR at the detector output, obtained via some signal processing technique, to compute the \( x \) element of the ordered pair mentioned above. For example, we can write the blind or group-blind detector output for User \( k \) at time \( i \) as \([2, \text{Eq. 47}]\)

\[
    z_k[i] = \w_k^H r[i] 
    \simeq \mu_k[i] b_k[i] + \nu_k[i] 
\]

where \( \nu_k[i] \sim \mathcal{N}_c(0, \theta_k^2[i]) \) and \( \mu_k[i] \) is the equivalent amplitude of the \( k \)-th user’s signal. We can estimate \( \mu_k[i] \) and \( \theta_k^2[i] \) via the sample mean and variance, respectively, of
\( \{ z_k[i] b_k[i] \} \) and compute the SINR at the output of the detector as [2, Eq. 78]

\[
\widehat{\text{SINR}}_k = \frac{\mu_k^2[i]}{2 \theta_k^2[i]}.
\]  

(6.59)

We can then generate ordered pairs as above, setting \( x \) equal to \( \widehat{\text{SINR}}_k / p_k \), compute a function estimate, and use this estimate as the mapping \( \mathcal{X} \).

Fig. 26. Scatter plot of “best” (but unavailable in practice) SINR slopes versus the corresponding SINR slopes of the MMSE detector, ignoring unknown users. The accompanying function estimate (solid line) is used as a mapping to obtain the \( \kappa \) parameters in Algorithm 2 from locally-available information, in this case the SINR slope of the MMSE detector, ignoring unknown users.

c. Algorithm Summary

We summarize UBPC for CDMA with blind or group-blind multiuser detectors via the following algorithm.
**Algorithm 5.** [Utility-Based Power Control for Blind or Group-Blind Multiuser Detectors (UBPC-BMUD)]

1. Define the locally-measurable quantity $x$ as described in Section b and create (offline) the mapping $X$.

2. Obtain utility functions and price parameters, $\{\zeta_k\}_{k=1}^{\hat{K}}$, from each known user. These are chosen using information about the traffic type [82].

3. Determine the set of functions $\{f^k(\cdot)\}_{k=1}^{\hat{K}}$ using the utility functions provided by the known users. Set $\eta = \frac{M}{\hat{N}}$.

4. Set $Q$ equal to the desired number of iterations and iterate the following procedure $Q$ times. It is assumed that $\{\hat{p}_k(0)\}_{k=1}^{\hat{K}}$ and $\{\text{SINR}_k(0)\}_{k=1}^{\hat{K}}$ are known.

For $q = 1, 2, \ldots, Q$

For $k = 1, 2, \ldots, \hat{K}$

(a) Estimate $\kappa_k(q)$ using the mapping $X$ and the appropriate locally-measurable quantity as defined in Step 1.

(b) Calculate

$$\widehat{\text{SINR}}_k(q) = f_k^{-1}\left(\frac{\zeta_k}{\kappa_k(q)}\right). \quad (6.60)$$

(c) If $\widehat{\text{SINR}}_k(q) < \text{SINR}_k$, set the transmit power for User $k$, $\hat{p}_k(q)$, to zero.

Otherwise set

$$\hat{p}_k^*(q) = \frac{\widehat{\text{SINR}}_k(q)}{\text{SINR}_k(q-1)} \hat{p}_k(q-1) \quad (6.61)$$

and choose the transmit power for User $k$ to be $\hat{p}_k(q) = \min\{\hat{p}_k^*(q), \eta/\kappa_k(q)\}$.  

After the transmit power is updated for all users, determine the new detector output SINRs, \( \{\text{SINR}_k(q)\}_{k=1}^K \). Note that in general, \( \text{SINR}_k(q) \neq \text{SINR}_k(q) \).

To develop a distributed, blind or group-blind implementation of this algorithm, we can use one of the blind channel estimation techniques discussed in Section 1 and we can generate the mapping \( \mathcal{X} \) using (6.56) or (6.57)-(6.59). We can estimate the output SINRs at the end of step 4 using (6.59).

The UBPC algorithm as developed in [82] was shown to be standard and, as a result, to have the following desirable properties:

1. There is a unique fixed power vector \( \mathbf{p}^* \).
2. The fixed point, \( \mathbf{p}^* \), is Pareto optimal (componentwise minimum).
3. UBPC converges from any initial power assignment to the unique fixed point \( \mathbf{p}^* \) in both synchronous and asynchronous cases. This implies convergence, for example, for situations in which some users perform power adjustments faster and execute more iterations than other users, and for situations in which updates are performed using outdated estimates of the interference caused by other users.

Assuming that UBPC-BMUD uses (6.56) to create the mapping \( \mathcal{X} \) and that the function estimate used for the mapping is monotonically increasing (as in Figure 26), one can easily prove positivity, monotonicity, and scalability for UBPC-BMUD using well known and easy to prove properties of the output SINR of exact MMSE detectors and of the functions \( \{ f_k^{-1}(\cdot) \}_{k=1}^K \).

One of the significant problems with [99] and many other power control algorithms is their behavior when the QoS requirements result in an infeasible system, i.e., a system in which no set of power assignments exists that simultaneously satisfies
all QoS requirements. In these situations, many power control algorithms can diverge, resulting in power assignments that grow without bound. UBPC-BMUD, however, inherits the graceful way in which UBPC deals with infeasible systems. In particular, when the environment becomes very hostile, there will be no gain (positive net utility) in transmitting for one or more users, hence they will turn their transmitters off. A more detailed discussion of infeasibility for UBPC (and, hence, UBPC-BMUD) may be found in [82].

3. Simulation Results

Figure 27 illustrates the evolution of the achieved SINR and transmit powers using UBPC-BMUD for synchronous CDMA over a non-fading AWGN channel. The processing gain is 15 and the total number of users varies as discussed below. The mapping in Figure 26 along with (6.56) was used to compute the $\kappa$ parameters. The AWGN power level is $5 \times 10^{-21}$ W/Hz, which produces a noise power of $\sigma^2 = 5 \times 10^{-15}$ W in a receiver of 1 MHz bandwidth. The distances between the users and the base station are constant and are chosen from a uniform distribution over the range 1 to 1000 meters. For these simulations, they are 347, 282, 177, 676, 372, 104, 492, 55, 980, and 902 meters for each of the 10 users, respectively. The price parameters for all users are set to unity. This is an uplink scenario, so the detector used is the group-blind hybrid multiuser detector and the SINR results reported in Figure 27 are computed using the SINR expression for group-blind detectors in [12, Eq. 51]. The signal subspace was obtained via an eigendecomposition of 256 received signal vectors. The distances are determined using estimates of the received signal powers and the (known) transmit powers. The received signal powers are estimated using the sample mean of $\left\{ z_k[i]b_k[i] \right\}_i$, where $z_k[i]$ is defined in (6.57). The results for the first 4 users are shown in the figure along with their utility functions. Notice that
the utility functions for User 1 and User 2 are steep, indicating that there is very little gain in operating at an SINR of more than 8 or 9 dB, respectively. The SINR results provided by UBPC-BMUD in this figure reflect these constraints. Steep utility functions are usually appropriate for voice-type traffic. The third and fourth users, however, have utilities that have higher center values and are less steep. This is more appropriate for data-type traffic. For the first frame (iterations 1 to 6), there are 8 total users and 6 known users. At iteration 6, two known users leave the system and at iteration 11, 4 known users are added to the system so that the total number of users is 10. Notice that at iteration 6, the transmit powers drop in response to the reduced interference as users leave the system. As users enter the system at iteration 11, the interference increases so much that the system becomes infeasible and User 4 must be turned off.

For the sake of comparison, we have repeated the simulations used to obtain Figure 27, except that we have replaced the new power control algorithm (UBPC-BMUD) with the unmodified utility-based power control algorithm from [82]. The results appear in Figure 28. Using the unmodified algorithm with blind or group-blind detectors leads to inaccurate power updates since the unmodified algorithm requires a linear relationship between each user’s SINR and transmit power; the actual relationship for blind and group-blind detectors is highly non-linear (see Figure 25). Note that there are non-zero data points for User 4 only at the beginning of each frame (iterations 1, 6, and 11). The data point at iteration 1 exists because all users begin the simulation with unit transmit power. The data points at the beginning of frames 2 and 3 represent attempts by the system to “restart” User 4 in light of the fact that users have left and/or entered the system. By the second iteration of each frame, however, the power control algorithm has turned off User 4. This is a waste of resources since it is known from Figure 27 that the system can support User 4 until
the third frame. This inefficiency is a direct result of the inaccurate power updates produced by the use of the unmodified power control algorithm with group-blind multiuser detection. A comparison with Figure 27 also reveals slower convergence relative to UBPC-BMUD.

E. (Group) Blind Adaptive Multiuser Detection with Transmitter Optimization

In this section, we develop a group-blind (blind as a special case) adaptive multiuser receiver that incorporates joint optimization of spreading codes and transmitter power. Essentially, we will incorporate the maximum eigenvector method (MEM) and UBPC-BMUD in an iterative algorithm in which MEM operates with constant powers delivered from UBPC-BMUD and UBPC-BMUD operates with constant spreading codes delivered by MEM from the previous iteration. This approach is similar in principle to the Lloyd-Max algorithm [100].

1. Receiver Structure and Implementation

Figure 29 contains a block diagram of the proposed transmitter/receiver. The information bit streams for the \( K \) users are mapped to BPSK or QPSK symbols and spread with (possibly non-binary and complex) spreading codes. These symbols pass through a fading multipath channel modelled by (6.12). The received signals are matched filtered and stacked to form \( r[i] \). The signal subspace used for channel estimation and for constructing the detector can be obtained via subspace tracking or an eigendecomposition of the received signal. We can estimate the channel using the blind sequential adaptive Kalman channel tracker in [94] along with the phase estimator in [13, Eq. 124]. The channel amplitudes can be estimated using estimates of the received signal power if \( C_0 \) and \( p_k, d_k \) are available for each known user. Infor-
Fig. 27. SINR and power evolution for a group-blind adaptive multiuser receiver for synchronous CDMA employing UBPC-BMUD over a non-fading AWGN channel in a dynamic environment in which users enter and leave the system. Data and utility functions are provided for the first 4 users.
Fig. 28. SINR and power evolution for a group-blind adaptive multiuser receiver for synchronous CDMA employing unmodified utility-based power control over a non-fading AWGN channel in a dynamic environment in which users enter and leave the system. Data are provided for the first 4 users.
mation from the subspace and channel estimators, along with utility functions, price parameters, and the mapping \( \mathcal{X} \), are used in an iterative transmitter optimizer that controls the power and spreading codes of the transmitter(s).

The feedback (receiver to transmitter) data rate should be considered in determining the feasibility of an algorithm of this type. Unfortunately, a full analysis is beyond the scope of this chapter, but we can see that the algorithm need only be executed when the system changes state, e.g., when users enter or leave the system, or when the channel changes significantly. The feedback data rate or the amount of information that must be sent back to the transmitter depends upon the processing gain, the number of known users, and whether or not we are allowing continuously varying codes. If the feedback data rate must be limited, we can quantize the power adjustments and spreading codes before they are fed back to the transmitter, with an accompanying loss in convergence rate. For these simulations, perfect feedback is assumed.

2. Simulation Results

Figure 30 shows the SINR and transmit power evolution for joint UBPC-BMUD and spreading code optimization for a fading multipath channel. The system parameters are described in Section 3 except for the following. The initial power assignments are 0 dB for all users. The utility functions for the first four users are as in Figure 27. Although this receiver is able to adapt in a dynamic channel environment, the channel has been fixed for these simulations. The total number of users is 9 and the processing gain is 15. Although this is an uplink scenario, SINR expressions do not yet exist for (estimated) group-blind detectors operating in fading multipath channels. As a result, the receiver used for this simulation is the subspace blind MMSE detector and the SINR results reported in Figure 30 are computed using (6.47). Spreading
Fig. 29. Blind ($\tilde{K} = 1$) or group-blind ($1 < \tilde{K} < K$) adaptive multiuser receiver with joint spreading code optimization and utility-based uplink power control. Data are provided for the first 4 users.
code optimization is performed after UBPC-BMUD iterations 5 and 9. Notice the significant drop in transmit powers after the first spreading code optimization. This is a result of the reduced interference caused by the new spreading codes. There is a smaller reduction in transmit powers after the second spreading code optimization and there is no significant change in transmit power levels after the third spreading code optimization (not shown).

Fig. 30. SINR and power evolution for a subspace blind adaptive multiuser receiver employing joint spreading code optimization and utility-based power control operating over a fading multipath channel. Data are provided for the first 4 users.
F. Conclusion

We have developed transmitter optimization techniques for blind and group-blind adaptive multiuser receivers. Specifically, we have presented a maximum-eigenvector-based approach to spreading code optimization and combined this with a utility-based power control algorithm in an iterative, Lloyd-Max type implementation that is appropriate for practical blind or group blind multiuser detectors. We have demonstrated that the spreading code optimization algorithm results in a significant improvement in received SINR and that the proposed power control algorithm, in contrast to existing utility-based schemes, can efficiently serve systems using blind or group-blind multiuser detectors. We have applied these algorithms to synchronous CDMA over non-fading AWGN channels and to asynchronous CDMA over fading multipath channels where one transmit and one receive antenna are employed. This work can easily be extended, however, to systems with multiple transmit and/or receive antennas through straightforward modifications to the signal and channel models. Future work will consider alternative methods of determining the slope parameters in distributed, blind settings.
CHAPTER VII

ADAPTIVE TRANSMITTER PRECODING FOR TIME DIVISION DUPLEX CDMA IN FADING MULTIPATH CHANNELS: STRATEGY AND ANALYSIS

A. Introduction

The demand for capacity and performance in multiple-access wireless systems has spurred the development of sophisticated signal processing techniques for signal reception [62, 101]. However, the goal of maintaining low cost and complexity, especially at the mobile unit, is as important as ever. As a result, researchers have recently begun investigating signal processing techniques that move computational complexity from the mobile unit to the base station, where it can be managed more efficiently. Generally speaking, these techniques involve some kind of transmitter-based multiuser interference cancellation at the base station (precoding) and simple linear processing, e.g., matched filtering, at the mobile unit. They are particularly appealing for time division duplex CDMA (TDD-CDMA) [102] since the same carrier is used for both uplink and downlink in different time slots. Hence, the downlink channel can be estimated at the base station using the uplink signals.

In [103], the authors considered transmitter precoding for synchronous CDMA over AWGN channels. They also present an extension to multipath channels, but a RAKE receiver is required and the channel is assumed perfectly known. A similar technique, pre-RAKE diversity combining, is investigated in [104]. However, RAKE reception is inherently sensitive to channel mismatch and performance is generally inferior to MMSE or decorrelator based multiuser interference rejection. The authors in [105] present a comparison of the techniques mentioned above using a typical vehicular channel. In [106], the authors consider transmitter precoding for multipath
fading channels but, in contrast to the present work, their prefilter is applied to the output of the spread spectrum encoder, rather than applying the filter first, followed by spreading.

In this chapter we develop a linear MMSE-based transmitter precoding strategy for the CDMA downlink in multipath fading channels. No RAKE receiver is required at the mobile unit, only matched filtering. Since we require channel information to construct optimal precoding filters, we implement blind channel estimation at the base-station, where complexity can be managed more efficiently. We also present a performance analysis using tools developed for the analysis of blind multiuser detection with blind channel estimation [13]. Finally, we develop an adaptive implementation that is able to adjust the precoding matrix as users enter and leave the system.

The remainder of this chapter is organized as follows. Section 2 describes the system under consideration. Section 3 develops the transmitter precoding strategy for CDMA over synchronous multipath channels. Section 4 presents a performance analysis. Section 5 discusses an adaptive implementation. Section 6 reports simulation results, and Section 7 concludes.

B. System Descriptions

1. Uplink Signal Model and Blind Channel Estimation

We consider a $K$-user discrete-time synchronous multipath CDMA system with no intersymbol interference (ISI). Such a system is realized either by neglecting the ISI when the multipath delay spread is small compared with the symbol interval, or by inserting guard intervals between symbols when the delay spread is large. The path delays are also assumed to be an integral number of chip periods and are known. We
first consider the chip-match filtered uplink signal received at the base station which, during the $i$-th symbol interval, can be written as

$$r[i] = \sum_{k=1}^{K} b_k[i] \sum_{l=1}^{L} s_{l,k} f_{l,k} + n[i]$$

(7.1)

where $L$ is the number of resolvable paths, $b_k[i]$ is the $i$-th symbol for the $k$-th user, $s_{l,k}$ and $f_{l,k}$ are, respectively, the delayed versions of the spreading waveform (with zero-padding when a guard interval is inserted) and the complex channel fading gain corresponding to the $l$-th path of the $k$-th user; $n[i] \sim \mathcal{N}_c(0, \sigma^2 I_N)$ is a complex white Gaussian noise vector. Note that $r[i], n[i] \in \mathbb{C}^N$ where $N$ is the processing gain. Denote

$$S_k \triangleq [s_{1,k} \ s_{2,k} \ \cdots \ s_{L,k}], \quad (7.2)$$

$$f_k \triangleq [f_{1,k} \ f_{2,k} \ \cdots \ f_{L,k}]^T. \quad (7.3)$$

Then (7.1) can be written as

$$r[i] = \sum_{k=1}^{K} S_k f_k b_k[i] + n[i]$$

(7.4)

$$= H b[i] + n[i]$$

(7.5)

where

$$H \triangleq [h_1 \ h_2 \ \cdots \ h_K], \quad (7.6)$$

$$b[i] \triangleq [b_1[i] \ b_2[i] \ \cdots \ b_K[i]]^T. \quad (7.7)$$

A block diagram of the uplink system appears in Figure 31.
Let the autocorrelation matrix of the received signal \( r[i] \) be

\[
C_r \triangleq E \{ r[i] r[i]^H \} = HH^H + \sigma^2 I_N
\]

(7.8)

\[
= U_s \Lambda_s U_s^H + \sigma^2 U_n U_n^H
\]

(7.9)

where (7.9) is the eigendecomposition of \( C_r \). Since the matrix \( H \) has full column rank \( K \), the matrix \( HH^H \) in (7.8) has rank \( K \). Therefore, in (7.9) \( \Lambda_s \) contains the \( K \) largest eigenvalues of \( C_r \); \( U_s \) contains the corresponding orthonormal eigenvectors; and \( U_n \) contains the \((N-K)\) orthonormal eigenvectors that correspond to the smallest eigenvalue, \( \sigma^2 \).

Suppose User 1 is the user of interest. Then since \( U_n^H h_1 = U_n^H S_1 f_1 = 0 \), we can estimate \( f_1 \) at the base station in the following way [93, 107, 108, 11]

\[
\hat{f}_1 = \arg \min_{\| f \| = 1} \| U_n^H S_1 f \|^2
\]

(7.10)

\[
= \arg \min_{\| f \| = 1} \hat{f}_1^H \left( S_1^H U_n U_n^H S_1 \right) \hat{f}_1
\]

(7.11)

\[
= \text{minimum eigenvector of } Q.
\]

(7.12)

Note that (7.12) specifies \( f_1 \) up to a scale and phase ambiguity and that, in practice, (7.12) can be implemented blindly in a batch or sequential adaptive manner. In batch mode, we simply replace the noise subspace parameters in (7.11) with parameters obtained from the eigendecomposition of the sample autocorrelation matrix of the received signal. In sequential adaptive mode, where we update the channel estimates at each time slot, we may employ a suitable subspace tracking algorithm and use the sequential Kalman filtering technique described in Section E.
2. Downlink Signal Model, Precoding and Receiver

The (downlink) signal transmitted from the base station during the $i$-th symbol interval can be written

$$x[i] = SMb[i]$$  \hspace{1cm} (7.13)

where

$$S \triangleq [s_1 \ s_2 \ \cdots \ s_K]$$  \hspace{1cm} (7.14)

is the matrix of spreading waveforms and $M \in \mathbb{C}^{K \times K}$ is a complex precoding filter which we will optimize in the following section. Throughout this chapter, we assume that the CDMA system is operating in the time division duplex mode, so that the downlink and uplink operate using the same carrier frequency in different time slots. We also assume that the time elapsing between uplink and downlink transmissions is sufficiently small compared to the coherence time of the channel that the channel impulse response is the same for the uplink and downlink. Then from (7.2) and (7.3), the received signal at User 1’s mobile unit can be written as

$$r_1[i] = \underbrace{\left[S_1 f_1 \ S_2 f_1 \ \cdots \ S_K f_1\right]}_{H_1} Mb[i] + n_1[i]$$  \hspace{1cm} (7.15)

where $S_1, S_2, \ldots, S_K$ contain shifted versions of their respective signature waveforms as in (7.2) except that the $L$ shifts are the same for each user’s waveform since all spreading codes have been transmitted over User 1’s downlink channel. Detection of the downlink information bits is accomplished via matched filtering of the received signal $r_1[i]$ with User 1’s signature waveform, $s_1$. Figure 32 contains a block diagram of the signal processing that takes place at the base station when an adaptive implementation is employed. We will see more details in Section E.
C. Transmitter Precoding for Synchronous Multipath CDMA

We seek to choose the precoding matrix $\mathbf{M}$ so as to provide the best downlink performance possible when the mobile units are constrained to the use of a matched filter receiver. We choose the minimum mean-square error criterion, so $\mathbf{M}$ is chosen to minimize

$$J = E \left\{ \left\| \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_K \end{bmatrix} \right\| \mathbf{s}_1^H \mathbf{r}_1 \\ \mathbf{s}_2^H \mathbf{r}_2 \\ \vdots \\ \mathbf{s}_K^H \mathbf{r}_K \right\|^2 \right\}$$

(7.16)

where we have dropped the time index for clarity. It is easy to see that

$$\begin{bmatrix} \mathbf{s}_1^H \mathbf{r}_1 \\ \mathbf{s}_2^H \mathbf{r}_2 \\ \vdots \\ \mathbf{s}_K^H \mathbf{r}_K \end{bmatrix} = \mathbf{H} \begin{bmatrix} \mathbf{s}_1^H \mathbf{n}_1 \\ \mathbf{s}_2^H \mathbf{n}_2 \\ \vdots \\ \mathbf{s}_K^H \mathbf{n}_K \end{bmatrix} = \mathbf{M} \mathbf{b} + \mathbf{v}.$$
Fig. 32. Adaptive precoding transmitter structure at the base station for the downlink signal.
Then

\[ J = E \{ \| b - \mathbf{H} M b - v \|^2 \}. \]  \hfill (7.18)

The following proposition gives the optimal precoding matrix.

**Proposition 3.** The choice of \( M \) that minimizes \( J \) is \( M = \mathbf{H}^{-1} \).

**Proof:** Although cumbersome, a direct proof be constructed via a complex analog to the proof in Appendix A of [103]. We offer the following simple alternative proof by contradiction. Suppose there exists a choice of \( M \), say \( M = M_0 \), that results in a smaller \( J \) than \( M = \mathbf{H}^{-1} \). Then

\[ E \{ \| b - \mathbf{H} M_0 b - v \|^2 \} < E \{ \| v \|^2 \}. \]  \hfill (7.19)

We may evaluate the left-hand side of (7.19) as

\[
E \{ \| b - \mathbf{H} M_0 b - v \|^2 \} = K - 2E \{ \text{Re} [b^H \mathbf{H} M_0 b] \} + E \{ b^H M_0^H \mathbf{H}^H \mathbf{H} M_0 b \} + E \{ v^H v \}. \]  \hfill (7.20)

Then (7.19) implies

\[
K - 2E \{ \text{Re} [b^H \mathbf{H} M_0 b] \} + E \{ b^H M_0^H \mathbf{H}^H \mathbf{H} M_0 b \} < 0. \]  \hfill (7.21)

However, the left-hand side of (7.21) is equal to \( E \{ \| b - \mathbf{H} M_0 b \|^2 \} \) which can never be less than zero. Hence, we have a contradiction. \( \square \)

Denote by \( \hat{\mathbf{H}}_i (1 \leq i \leq K) \) the matrix \( \mathbf{H}_i \) where the channel \( \mathbf{f}_i \) has been replaced with the blind estimate \( \hat{\mathbf{f}}_i \) obtained from (7.12). Then we may form an initial blind
estimate of $M$ at the base station as

$$\hat{M} = \begin{bmatrix} \bar{s}_1^H \hat{H}_1 \\ \bar{s}_2^H \hat{H}_2 \\ \vdots \\ \bar{s}_K^H \hat{H}_K \end{bmatrix}^{-1}. \quad (7.22)$$

There remain amplitude and phase ambiguities in $\hat{M}$ that are addressed in the following sections.

**Remark:** We should expect the performance of transmitter-based multiuser detection to be somewhat inferior to traditional receiver-based approaches since the transmitter precoding filter must satisfy more requirements than the detector in a receiver-based approach. More specifically, notice that the objective function in (7.16) requires that the choice of $M$ maximize performance for all users *simultaneously*. Choosing $M$ to minimize an objective of the form

$$J_0 = \mathbb{E} \left\{ |b_1 - s_1^H r_1|^2 \right\} \quad (7.23)$$

$$J_0 = \mathbb{E} \left\{ |b_1 - s_1^H (H_1 M b[i] + n_1[i])|^2 \right\} \quad (7.24)$$

could provide better downlink performance for User 1 than $J$ of (7.16), but at the expense of the other users. Hence $J'$ is not an acceptable cost function. In this sense, the simultaneity requirement means that transmitter precoding has fewer degrees of freedom for combatting interference than does MMSE or decorrelating receiver-based multiuser detection.

**D. Performance Analysis**

In [13, 12, 15, 109, 17], the authors developed analytical tools to investigate the performance of blind and group-blind linear MMSE multiuser detection. In this section,
we adapt these tools to the analysis of transmitter precoding with blind channel estimation. In particular, we will derive signal-to-interference-plus-noise (SINR) and BER expressions that take residual multiple-access interference and channel estimation error into account. In Section 6, we will compare these expressions to simulation results.

Notice that the estimate $\hat{M}$ given by (7.22) is not a consistent estimate of $M$ because of the unknown phase and scaling factors. However, there is a diagonal matrix $\Phi$ so that $\hat{M}\Phi^{-1}$ is a consistent estimate. The matrix $\Phi$ is of the form

$$\Phi \triangleq \text{diag}(\|f_1\|e^{j\phi_1}, \|f_2\|e^{j\phi_2}, \ldots, \|f_K\|e^{j\phi_K})$$

(7.25)

where $\phi_k$, $k = 1, \ldots, K$ are phase factors that depend on how the estimation is implemented. With this in mind, we state the following result, which is proved in the appendix.

**Theorem 1.** Let $\hat{M}$ be given by (7.22), and let $b$ be i.i.d. QPSK symbols independent of $\hat{M}$. Then

$$\sqrt{M} \left( [\hat{M}\Phi^{-1} - M]b \right) \to N_c(0, C_m) \text{ in distribution as } M \to \infty$$

with

$$C_m = \mathcal{H}^{-1} D \mathcal{H}^{-H}$$

(7.26)

where the diagonal elements of $D$ are given by

$$[D]_{i,i} = \beta_i \sum_{k=1}^{K} \sum_{l=1}^{K} [\mathcal{H}^{-1}\mathcal{H}^{-H}]_{k,l}s_i^H S_k Q_s S_i^H s_i$$

(7.27)

and

$$\beta_i \triangleq \sigma^2 h_i^H U_s A_s (A_s - \eta I_K)^{-2} U_s^H h_i,$$

(7.28)
while the off-diagonal elements can be ignored with good accuracy. Here $Q_i^\dagger$ denotes the Moore-Penrose generalized inverse \cite{92} of the matrix $Q_i \triangleq S_i^H U_n U_n^H S_i$.

The SINR at the output of the matched filter for User 1 is given by \cite{13, 12}

\[
\text{SINR} \triangleq \frac{\left| E \left\{ s_i^H r_1[i] | b_1[i] \right\} \right|^2}{E \{ \text{Var} (s_i^H r_1[i] | b_1[i]) \}}.
\] (7.29)

Now suppose that the phase and amplitude factors in $\Phi$ have been determined. Write the estimated matrix, $\hat{M}$, as $\hat{M} \Phi^{-1} = M + \Delta M$, where $\Delta M$ is the estimation error. Dropping the time index for clarity, the received signal can then be written as

\[
 r_1 = s_1^H r_1
 = (s_1^H H_1) \hat{M} \Phi^{-1} b + s_1^H n_1
 = (s_1^H H_1) M b + (s_1^H H_1) \Delta M b + s_1^H n_1
 = (s_1^H H_1) [M]_{:,1} b_1 + (s_1^H H_1) [M]_{:,2:K} [b]_{2:K} + (s_1^H H_1) \Delta M b + s_1^H n_1
\] (7.33)

where the notation $[M]_{:,2:K}$ indicates the matrix composed of columns 2 through $K$ of the matrix $M$. According to Theorem 1, for large $M$ the third term in (7.33) is also Gaussian distributed (independent of the other terms) with variance

\[
\nu_1^2 = \frac{1}{M} (s_1^H H_1) C_m H_1^H s_1.
\] (7.34)

Since $M$ represents an MMSE detector, we can also make the approximate assumption that the multiple-access interference is Gaussian distributed \cite{21}. We can therefore calculate the BER via a single $Q$-function as

\[
P_b(e) \cong Q(\sqrt{\text{SINR}})
\] (7.35)

with

\[
\text{SINR} = \frac{\left| (s_1^H H_1) [M]_{:,1} \right|^2}{\sum_{k=2}^{K} \left| (s_1^H H_1) [M]_{:,k} \right|^2 + \sigma^2 \| s_1 \|^2 + \frac{1}{M} (s_1^H H_1) C_m H_1^H s_1}.
\] (7.36)
Notice that the first term in the denominator of the SINR expression is due to residual multiple-access interference. The second term is the ambient noise, and the third term is due to the channel estimation error.

E. Adaptive Implementation

In this section we present an adaptive implementation of the transmitter precoding strategy discussed in Section C that updates the precoding matrix $M$ at each time slot. This sequential updating allows the implementation to adapt as the channel changes and as users enter and leave the system. A block diagram of the signal processing at the base station appears in Figure 32. Note that we have suppressed the signal processing necessary for detection of the uplink bits. The uplink signal received at the base station is used in a signal subspace tracker, along with the known spreading codes of all users, to construct channel estimates by solving (7.40). Recall that since we are assuming TDD mode, the uplink channel estimates also serve as downlink channel estimates that can be used to construct $M$. As previously mentioned, these channel estimates have amplitude and phase ambiguities. Since nearly all cellular CDMA systems employ power control, it is likely that the base station has some knowledge of each users’ transmit power. This information, coupled with estimates of the received power, can be used to estimate the channel amplitude for each user. More specifically, let the diagonal matrices $A = \text{diag}(\alpha_1, \alpha_2, \ldots, \alpha_K)$ and $P = \text{diag}(\sqrt{p_1}, \sqrt{p_2}, \ldots, \sqrt{p_K})$ contain the unknown channel amplitudes and the known uplink transmit powers, respectively. Also define $\tilde{H}$ such that $H = \tilde{H}AP$, where we have separated the transmit powers and channel amplitudes from $H$. We propose an estimator based on the following fact, which is proved in the appendix.
Proposition 4.

\[ A = \left[ \bar{H}^H U_s (\Lambda_s - \sigma^2 I_K)^{-1} U_s^H \bar{H} P^2 \right]^{-\frac{1}{2}} \]  

(7.37)

where \( U_s, \Lambda_s \) are signal subspace components derived from an eigendecomposition of the autocorrelation matrix, \( C_r \), of the received signal, as in (7.8) and (7.9).

We may obtain an estimate, \( \hat{A} \), of \( A \) by replacing \( \bar{H}, U_s, \Lambda_s \) and \( \sigma^2 \) of (7.37) with their respective estimates obtained from subspace tracking and the solution to (7.40). The subspace tracker we have chosen to use for this adaptive implementation is NAHJ-FST (noise averaged Hermitian-Jacobi fast subspace tracking), which has complexity \( O(NK) \) floating operations per user per bit and which performs close to the lower bound for all SVD-based subspace trackers [63]. The application of NAHJ-FST to the current tracking problem is a straightforward modification of [63] and will not be discussed in detail. The channel phase ambiguity can be circumvented by the use of differential encoding and decoding of the data. After channel and amplitude estimation, the amplitude corrected channel information is then used, along with the known spreading codes, to construct the precoding matrix \( \hat{M} \). Finally, the downlink information bits are differentially encoded and filtered with \( \hat{M} \) before spreading and transmission. At the mobile unit, matched filtering and differential detection are performed to obtain estimates of the downlink information bits.

1. Blind Sequential Adaptive Channel Estimation

Here we describe the adaptive channel estimator used in Figure 32. Consider User 1 the user of interest and denote by \( z[i] \) the projection of received uplink signal \( r[i] \)
onto the noise subspace, i.e.,

\[ z[i] = r[i] - U_suHs r[i] \quad (7.38) \]

\[ = U_n U^H_n r[i]. \quad (7.39) \]

Since \( z[i] \) lies in the noise subspace, it is orthogonal to any signal in the signal subspace and, in particular, it is orthogonal to \( S_1f_1 \). Hence \( f_1 \) is the solution to the following constrained optimization problem:

\[
\min_{f_1 \in \mathbb{C}^L} E \left\{ \left\| z[i]^H S_1 f_1 \right\|^2 \right\} \\
= \min_{f_1 \in \mathbb{C}^L} E \left\{ \left\| (S_1^H z[i])^H f_1 \right\|^2 \right\} \quad \text{s.t.} \quad \|f_1\| = 1. \quad (7.40)
\]

In order to obtain a sequential algorithm to solve the above optimization problem, we write it in the following (trivial) state space form

\[
f_1[i + 1] = f_1[i], \quad \text{state equation}
\]

\[
0 = \left[ S_1^H z[i] \right]^H f_1[i], \quad \text{observation equation.}
\]

Denote \( x[i] \triangleq S_1^H z[i] \). Then the standard Kalman filter can be applied to the above system as

\[
k[i] = \Sigma[i - 1] x[i] \left( x[i]^H \Sigma[i - 1] x[i] \right)^{-1}, \quad (7.41)
\]

\[
f_1[i] = f_1[i - 1] - k[i] \left( x[i]^H f_1[i - 1] \right) / \| f_1[i - 1] - k[i] \left( x[i]^H f_1[i - 1] \right) \|, \quad (7.42)
\]

\[
\Sigma[i] = \Sigma[i - 1] - k[i] x[i]^H \Sigma[i - 1]. \quad (7.43)
\]

with the initial condition \( \Sigma[0] = I_L \).

2. Algorithm Summary

We may summarize the adaptive implementation at the base station as follows.
Algorithm 6. [Sequential adaptive transmitter precoding for synchronous multipath CDMA]

1. Using a suitable signal subspace tracking algorithm, e.g. NAHJ-FST, update the signal subspace components \( U_s[i], \Lambda_s[i], \) and \( \sigma^2[i] \) at each time slot \( i \) using the uplink signals.

2. Track the channels \( \{f_k\}_{k=1}^K \) as follows:

\[
\begin{align*}
z[i] &= r[i] - U_s[i]U_s[i]^H r[i], \\
x[i] &= S_k^H z[i], \\
k[i] &= \Sigma[i - 1] x[i] (x[i]^H \Sigma[i - 1] x[i])^{-1}, \\
f_k[i] &= f_k[i - 1] - k[i] (x[i]^H f_k[i - 1] - k[i] (x[i]^H f_k[i - 1])) / \| f_k[i - 1] - k[i] (x[i]^H f_k[i - 1]) \|, \\
\Sigma[i] &= \Sigma[i - 1] - k[i]^2 x[i]^H \Sigma[i - 1].
\end{align*}
\]

3. Calculate the channel amplitudes via (7.37) using the channel estimates, the signal subspace parameters, the known spreading codes, and the known transmit powers.

4. Using (7.17) and the information from steps 1-3, calculate \( H \) and set \( M = H^{-1} \).

5. Differentially encode the downlink bit streams for each user to form \( b[i] \).

6. Transmit the precoded downlink signal \( x[i] = SMb[i] \).

7. Perform matched filtering and differential detection at the mobile units.

F. Simulation Results

1. Analytical Performance Versus Simulated Performance

Here we compare the BER expression in (7.36) to simulation results. We will also compare the analytical and simulated performance to conventional receiver-based subspace blind MMSE multiuser detection. The simulated system is a QPSK modulated...
synchronous multipath CDMA system as described in Section B, i.e.,

\[ b_k[i] \in \left\{ \frac{1 + j}{\sqrt{2}}, \frac{1 - j}{\sqrt{2}}, \frac{-1 + j}{\sqrt{2}}, \frac{-1 - j}{\sqrt{2}} \right\}. \]

The spreading codes for each user are \( m \)-sequences of length 15 and their shifted versions. The number of users in the system is 10. The number of paths induced by each user’s channel is \( L = 4 \), and the channel is kept constant for all simulations. Blind channel estimation is performed on the uplink signals using (7.12), where the exact noise subspace is replaced by an estimated noise subspace obtained from an eigendecomposition of the received signal, as in [13]. The frame length used for estimating the channel (and the precoding filter) is either \( M = 200 \) or \( M = 2000 \), as noted on the figures. As mentioned previously, there is an amplitude and phase ambiguity inherent in (7.12). For now they are assumed known. These ambiguities are resolved in the adaptive implementation discussed next.

Figure 33 illustrates the best and worst BER performance among the 10 users using (7.35) for the analytical performance of transmitter precoding and equation (98) of [13] for the analytical performance of receiver-based subspace blind MMSE multiuser detection (denoted rx-mud:min and rx-mud:max in the figure). The SNR is defined as \( E_b/(2\sigma^2) \). Notice that there is a very good match between the simulated and analytical performance results for transmitter precoding. The previously mentioned performance penalty of transmitter precoding relative to receiver-based multiuser detection is also evident. The number of signal samples used to estimate the channels and, hence, \( M \) is 2000.

The simulation parameters for Figure 34 are identical to those for Figure 33 except that the number of signal samples used to estimate \( M \) has been reduced from 2000 to 200. Although the performance of subspace blind MMSE multiuser detection is still superior, the error floor that appears its performance does not appear in the
performance of transmitter precoding. It is clear that an error floor must exist for both techniques since the detector or precoder is estimated from noisy received signals, but this result suggests that the performance of transmitter precoding may degrade more gracefully than that of subspace blind MMSE multiuser detection when the number of signal samples available diminishes.

Figure 35 contains plots of the analytical and simulated SINR versus the number of signal samples used to construct the precoder matrix for the best performing user. The analytical values are obtained from the SINR expression in (7.36). The simulated for values are taken from

\[
\text{SINR}_k = \frac{|E \{z_k[i] \mid b_k[i]\}|^2}{E \{\text{Var} [z_k[i] \mid b_k[i]]\}} \tag{7.44}
\]

where the expectations are replaced with time averaging and where \(z_k[i]\) is the decision statistics for user \(k\) at time slot \(i\). This figure also contains an indication of the theoretical SINR when perfect channel information is available, i.e., for \(M = \infty\). As was the case with the BER plots, there is good agreement between the simulated and theoretical results.

2. Performance of Adaptation Implementation

Here we examine the simulated performance of the adaptive implementation of transmitter precoding discussed in Section E. We consider the same \(K\)-user synchronous multipath CDMA system used in the previous simulation results, except that the modulation used is BPSK instead of QPSK. The channel is kept constant during all simulations and the SNR is fixed at 18dB.

Figure 36 contains the a plot of the average SINR for the first four users versus the time index. Since we are using differential detection, the decision statistic for
Fig. 33. Comparison of analytical and simulated performance results for transmitter precoding. Also presented for comparison is the analytical performance of (receiver-based) subspace blind MMSE multiuser detection. The number of signal samples used to construct the precoders and detectors is 2000.
Fig. 34. Comparison of analytical and simulated performance results for transmitter precoding. Also presented for comparison is the analytical performance of (receiver-based) subspace blind MMSE multiuser detection. The number of signal samples used to construct the precoders and detectors is 200.
Fig. 35. Analytical and simulated SINR results for transmitter precoding. The SNR is fixed at 18dB.
detecting bit $i$ of user $k$ is

$$z_k[i] = \text{Re} \left\{ (s_k^H r_k[i - 1])^H (s_k^H r_k[i]) \right\}$$  \hspace{1cm} (7.45)

and the SINR is defined here as

$$\text{SINR} = \frac{|E \{ z_k[i] \mid d_k[i] - 1 \} d_k[i] \} |^2}{E \{ \text{Var} [z_k[i] \mid d_k[i - 1] d_k[i]] \}}$$  \hspace{1cm} (7.46)

where the $d_k[i]$ represents the $i$-th differentially coded bit for user $k$. For these simulations, the expectation is replaced with time averaging. During the first 1000 time slots, there are 7 users in the system. At time slot 1001, 3 users are added to the system and at iteration 2001, 6 users are removed from the system. We see that the adaptive system rapidly adjusts (within 500 time slots) when users leave and enter the system.

Figure 37 contains a plot of the bit-error-rate versus SNR. The system has been allowed 500 received signal samples to reach steady state before errors are accumulated. It is tempting to compare this figure to Figures 33 and 34 but we must keep in mind that the blind adaptive implementation uses BPSK instead of QPSK and differential encoding/decoding rather than coherent detection.

G. Conclusions

In this chapter, we have developed a transmitter precoding strategy for the downlink of a CDMA system in fading multipath channels. The technique presented precodes the downlink bits before spreading and transmission and is optimal in the mean-squared-error sense. We have presented a performance analysis using tools developed for the analysis of (receiver-based) blind multiuser detection and we have developed an adaptive implementation of the precoding strategy that is able to adjust the pre-
Fig. 36. Adaptation performance of adaptive transmitter precoding.
Fig. 37. Steady-state performance of adaptive transmitter precoding.
coding matrix as users enter and leave the system. Simulation results indicate a very close match between simulated and analytical performance. We also see through simulation that the adaptive implementation is able to quickly and successfully adapt as users enter and leave the system. Future work will address asynchronous multipath channels and cases where the uplink and downlink channels are not identical, but only statistically highly correlated.
CHAPTER VIII

CONCLUSION

The focus of the this work has been signal processing for the mitigation of interference in CDMA systems. We have categorized this research into four topical areas: turbo processing, space-time processing, transmitter optimization, and low-complexity blind adaptive multiuser detection.

In the area of turbo processing, we have developed a low-complexity turbo equalizer for severe intersymbol interference (ISI) channels that can completely eliminate ISI for moderate and fast fading channels. We also derived a turbo group-blind multiuser receiver that is appropriate for uplink CDMA in which the base station has knowledge of spreading codes of the users within the cell, but no knowledge of the codes of users outside the cell.

In the area of low-complexity blind adaptive multiuser detection, we have developed a new high-performance, low-complexity subspace tracking algorithm and used it in conjunction with a blind adaptive Kalman channel estimator to construct a low-complexity blind adaptive group-blind multiuser detector for CDMA systems operating over multipath fading channels.

In the area of space-time processing, we considered the performance of linear space-time multiuser detection using multiple transmit and receive antennas for CDMA systems in multipath fading environments. We saw that the space-time multiuser receiver has significant advantages over linear diversity combining, including better performance, lower complexity, and higher capacity.

In the area of transmitter optimization, we have investigated joint power control and spreading code optimization for CDMA systems that utilize blind or group-blind multiuser detectors. We made use of recent analytical results concerning the
performance of these detectors in the development of the power control algorithm. We also considered a transmitter precoding strategy (transmitter-based multiuser detection) for CDMA systems that allows for simple matched filtering at the mobile units. We saw that transmitter precoding is a reasonable alternative to conventional receiver-based multiuser detection when minimizing complexity at the mobile unit is a priority.

Ideas for future work include extending the transmitter precoding strategy and analysis to asynchronous CDMA and to situations in which the uplink and downlink channels are not identical, but only highly correlated. Another interesting direction for future research is an investigation of the relationships between transmitter precoding, space-time coding, and spreading code optimization. Most existing research analyzes and develops these techniques separately, but it is clear that a unified study might lead to useful design insights. Upper network layer functionalities such as access control, call admission control, automatic repeat request (ARQ), and power control also represent a fertile area of research because it is not currently known how these upper-layer functionalities should be designed to take into account the new advanced signal processing techniques developed for the physical layer (e.g., turbo processing, space-time processing).
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APPENDIX A

PROOF OF THEOREM 1

In this appendix, we prove Theorem 1 of Chapter VII. We begin with a lemma.

Lemma 1 (Corollary 1 in [13]). Let \( f_1 \) be the true channel of user 1, and let \( \hat{f}_1 \) be the channel estimate given by (7.12). Then there exists a phase factor \( e^{j\phi} \), such that

\[
\sqrt{M} \left( \hat{f}_1 - \|f_1\|^{-1} e^{j\phi} f_1 \right) \to \mathcal{N}_c \left( 0, \beta_1 \|f_1\|^{-2} Q_1^t, 0 \right) \text{ in distribution as } M \to \infty,
\]

with

\[
\beta_1 \triangleq \sigma^2 h_1^H U_s A_s (A_s - \eta I_K)^{-2} U_s^H h_1, \tag{A.2}
\]

\[
Q_1 \triangleq S_1^H U_n U_n^H S_1 \tag{A.3}
\]

and where the notation \( \mathcal{N}_c(\mu, C, \bar{C}) \) indicates a complex Gaussian distribution with mean \( \mu \), Hermitian covariance matrix \( C \), and symmetric covariance matrix \( \bar{C} \).

Proof of Theorem 1: Let \( \Delta M \) denote the differential of the function \( \hat{M} \) [12]. Then

\[
\Delta M = -\mathcal{H}^{-1} \Delta \mathcal{H} \mathcal{H}^{-1}, \tag{A.4}
\]

\[
\Delta \mathcal{H} = \begin{bmatrix}
    s_1^H \Delta H_1 \\
    s_2^H \Delta H_2 \\
    \vdots \\
    s_K^H \Delta H_K 
\end{bmatrix}, \tag{A.5}
\]

\[
\Delta H_k = \begin{bmatrix}
    S_1 \Delta f_k \\
    S_2 \Delta f_k \\
    \cdots \\
    S_K \Delta f_k
\end{bmatrix}. \tag{A.6}
\]

Now each \( \Delta f_k \) is asymptotically circularly Gaussian distributed by Lemma 1. It follows that the same holds for \( \Delta M \), and since \( b \) is independent of \( \hat{M} \), this is also
true for $\Delta M \mathbf{b}$, and the Theorem follows. It remains to calculate $C_m$. To this end notice that

$$
\Delta \mathbf{H} = 
\begin{bmatrix}
    s_1^H S_1 \Delta f_1 & s_1^H S_2 \Delta f_2 & \cdots & s_1^H S_K \Delta f_1 \\
    s_2^H S_1 \Delta f_2 & s_2^H S_2 \Delta f_2 & \cdots & s_2^H S_K \Delta f_2 \\
    \vdots & \vdots & \ddots & \vdots \\
    s_K^H S_1 \Delta f_K & s_K^H S_2 \Delta f_K & \cdots & s_K^H S_K \Delta f_K 
\end{bmatrix}.
$$

(A.7)

Then

$$
\begin{align*}
    ME \{ \Delta M \mathbf{b} \Delta M^H \} &= ME \{ \Delta M \Delta M^H \} \\
    &= \mathbf{H}^{-1} ME \{ \Delta \mathbf{H} \mathbf{H}^{-H} \Delta \mathbf{H}^H \} \mathbf{H}^{-H}. 
\end{align*}
$$

(A.9)

Here we have

$$
M[D]_{i,j} = ME\left\{ \begin{bmatrix}
    s_1^H S_1 \Delta f_1 & s_1^H S_2 \Delta f_1 & \cdots & s_1^H S_K \Delta f_1 \\
    s_2^H S_1 \Delta f_2 & s_2^H S_2 \Delta f_2 & \cdots & s_2^H S_K \Delta f_2 \\
    \vdots & \vdots & \ddots & \vdots \\
    s_K^H S_1 \Delta f_K & s_K^H S_2 \Delta f_K & \cdots & s_K^H S_K \Delta f_K 
\end{bmatrix} \mathbf{h}_j^H \right\}
$$

$$
= \sum_{k=1}^{K} \sum_{l=1}^{K} [\mathbf{H}^{-1} \mathbf{H}^{-H}]_{k,l} s_i^H S_k \Delta f_i \Delta f_j^H ME \{ \Delta f_i \Delta f_j^H \} S_l^H s_j.
$$

(A.11)

We can disregard crosscorrelation between different channel estimates and, therefore, we have $[\mathbf{X}]_{i,j} \approx 0$ for $i \neq j$, and for $i = j$ we have, by Lemma 1,

$$
ME \{ \Delta f_i \Delta f_i^H \} = \beta_i \| f_i \|^{-2} Q_i^i.
$$

(A.12)

If we substitute (A.12) into (A.11) and compensate for the amplitude and phase factors, then (A.9) is equal to (7.26) and the proof is complete. \qed
APPENDIX B

PROOF OF PROPOSITION 1

In this appendix, we prove Proposition 1 of Chapter V.

By (5.11) and (5.20) it suffices to show that

$$\left[ \tilde{R}^{-1} \right]_{1,1} \leq \frac{1}{E_1} \left[ R^{-1} \right]_{1,1}. $$

We make use of the following facts. Denote $A_{i,j}$ as the submatrix of $A$ obtained by striking out the $i$-th row and the $j$-th column. Then it is known that

$$A \succeq 0 \implies A - \frac{1}{[A^{-1}]_{k,k}} e_k e_k^T \succeq 0. \quad \text{(B.1)}$$

It is also known that

$$A \succeq 0, \; B \succeq 0 \implies A \circ B \succeq 0. \quad \text{(B.2)}$$

Assuming $R \succeq 0$ and $Q \overset{\Delta}{=} H^H H \succeq 0$, and using the above two results, we have

$$0 \leq \det \left[ \left( R - \frac{1}{[R^{-1}]_{1,1}} e_1 e_1^T \right) \circ Q \right]$$

$$= \det \left[ \tilde{R} - \frac{E_1}{[R^{-1}]_{1,1}} e_1 e_1^T \right] \quad \text{(B.3)}$$

$$= \det \tilde{R} \left( 1 - \frac{E_1}{[R^{-1}]_{k,k}} e_k^T \tilde{R}^{-1} e_1 \right) \quad \text{(B.4)}$$

$$= \det \tilde{R} - \frac{E_1}{[R^{-1}]_{1,1}} \det \tilde{R}_{1,1}, \quad \text{(B.5)}$$

where (B.3) follows from the fact that $\tilde{R} \overset{\Delta}{=} R \circ Q$ and $(e_1 e_1^T) \circ Q = E_1 e_1 e_1^T$; (B.4) follows from the matrix identity

$$\det (A + BCD) = \det A \det C \det (C^{-1} + DA^{-1}B); \quad \text{(B.6)}$$
and (B.5) follows from

\[ e_1^T \tilde{R}^{-1} e_1 = \left[ \tilde{R}^{-1} \right]_{1,1} = \frac{\det \tilde{R}_{1,1}}{\det \tilde{R}}. \]  

(B.7)

Hence we have

\[ \frac{1}{\left[ \tilde{R}^{-1} \right]_{1,1}} = \frac{\det \tilde{R}}{\det \tilde{R}_{1,1}} \geq \frac{E_1}{\left[ R^{-1} \right]_{1,1}}. \]  

(B.8)
APPENDIX C

PROOF OF PROPOSITION 4

In this appendix, we prove Proposition 4 of Chapter VII.

Since

\[ \hat{H} A^2 P^2 \hat{H}^H + \sigma^2 I_N = U_s \Lambda_s U_s^H + \sigma^2 U_n U_n^H \]  
\[ = HH^H + \sigma^2 I_N \]  
\[ = C_r, \]  

it is easy to see that \( \hat{H} A^2 P^2 \hat{H}^H = U_s (\Lambda_s - \sigma^2 I_N) U_s^H \). It can also be verified using the definition of the Moore-Penrose generalized matrix inverse that \( (\hat{H} A^2 P^2 \hat{H}^H)^\dagger = (\hat{H}^H)^\dagger A^{-2} P^{-2} \hat{H}^\dagger \). Then

\[ (\hat{H}^H)^\dagger A^{-2} P^{-2} \hat{H}^\dagger = U_s (\Lambda_s - \sigma^2 I_K)^{-1} U_s^H \]  

and, solving for \( A^{-2} \), we have

\[ A^{-2} = \hat{H}^H U_s (\Lambda_s - \sigma^2 I_K)^{-1} U_s^H \hat{H} P^2 \]  

and the proposition follows. \( \square \)
VITA

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