Linear Precoding and User Scheduling for Downlink TDD-CDMA

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Daryl Reynolds § Anders Høst-Madsen ¶

Abstract

In this paper, we first compare two classes of linear interference suppression techniques for downlink TDD-CDMA systems, namely, linear multiuser detection methods (receiver processing) and linear precoding methods (transmitter processing). For the linear precoding schemes, we assume that the channel state information (CSI) is available only at the transmitter but not at the receiver. We propose several precoding techniques and the corresponding power control algorithms. The performance metric used in the comparisons is the total power required at the transmitter to achieve certain QoS at the receiver. Our results reveal that in general multiuser detection and precoding offer similar performance; but in certain scenarios, precoding can bring a substantial performance improvement. These results motivate the use of precoding techniques to reduce the complexity of the mobile terminals (only a matched-filter to the own spreading sequence is required without CSI). Moreover, it is shown that the proposed chip-wise linear MMSE precoding method is optimal in the sense that it requires the minimum total transmitted power to meet a certain receiver QoS performance. On the other hand, the CSI at the transmitter can facilitate efficient user scheduling. We therefore further develop low-complexity user allocation algorithms based on the proposed linear precoding techniques.

Keywords: Linear multiuser detection, linear precoding, downlink CDMA, user allocation, power control.

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1 Introduction

In the uplink CDMA wireless systems, it is assumed that the base station has access to all users’ channel state information (CSI) and spreading signatures; and multiuser detection (MUD) has been shown to be an effective way to combat interference and increase data throughput [12]. For the downlink, on the other hand, one can transfer the signal processing for interference suppression from the mobile receiver to the transmit base station by using precoding techniques. This is feasible if the base station has access to the CSI of all active mobile units, e.g., in time-division duplex (TDD) systems where the base station can exploit the channel reciprocity if the time difference between uplink and downlink transmission is shorter than the channel coherence time, or by using channel prediction techniques [1]. The simplest precoding method is pre-RAKE [2], which mitigates the multipath interference without considering the multiuser interference (MUI). Linear precoding techniques to remove the MUI and multipath interference were proposed in [13]. Nonlinear precoding techniques have been shown to offer superior performance although they complicate the receiver and the transmitter, since a modulo operation (which in general depends on the CSI) has to be implemented at both sides of the communication link [3, 9, 14]. Note that most work on linear precoding assumes that each user implements a RAKE receiver and hence assumes the knowledge of CSI at the receiver [13, 14].

In this paper, we consider linear precoders with ultra-simple receivers, i.e., only a fixed matched-filter to the spreading sequence without CSI. We propose several bit-wise and chip-wise linear precoders and the corresponding power control algorithms to meet certain performance at the receiver. We also consider the performance comparisons between linear precoding and linear MUD. The comparison metric is the total required power at the transmitter to achieve a minimum QoS requirement at each of the receivers. Our results show that linear precoding offers similar performance as linear MUD in most cases; but in some specific cases, linear precoding is more effective. Moreover, the proposed linear precoding techniques with only the matched-filter (to the spreading sequence) at the receiver can outperform the linear precoder with a RAKE receiver (i.e., with CSI at the receiver) proposed in [13]. These results motivate the use of linear precoding techniques in the downlink of TDD-CDMA systems. Among the advantages of using linear precoding we have:

- Receiver terminals are limited to a fixed matched-filter to the own spreading sequence. This translates into a power consumption reduction and decrease in price of the ter-
minals since they do not have to perform sophisticated signal processing for channel estimation and interference mitigation. Note that variations in channel conditions and number of active users in the network do not affect the receiver operations.

- Less amount of control data is required in the precoding solution. The reason is that in MUD, every user requires to know the own channel response plus the spreading sequences and the CSI of all other active users in the network. Moreover, mobile units do not need to be informed when users are added to (or removed from) the network.

- Power control is easy to implement with linear precoding since the receiver has information about the quality of each link and it does not require extra feedback information. Note that MUD requires a feedback link to find the power loading value assigned to each user.

- User scheduling based on the knowledge of CSI can be implemented jointly with linear precoding to increase the system throughput.

The remainder of this paper is organized as follows. In Section 2 we briefly summarize two well-known linear MUD methods and the corresponding power control algorithms. In Section 3 we propose several forms of linear precoding schemes and discuss their properties. In Section 4 we present simulation comparisons between linear MUD and linear precoding. In Section 5 we discuss low-complexity user scheduling algorithms. Finally, Section 6 concludes the paper.

## 2 Linear MUD Methods

We consider a $K$-user discrete-time synchronous multipath CDMA system. Define $b_k[i]$ from a constellation $\mathcal{A}$ as the symbol of the $k$-th user transmitted during the $i$-th symbol interval with $\mathbb{E}\{|b[i]|^2\} = 1$ and $b[i] = [b_1[i], ..., b_K[i]]^T$. Denote $N$ as the spreading factor and $s_k = [s_{k,1}, ..., s_{k,N}]^T$ as the normalized spreading waveform of the $k$-th user. Then, the signal transmitted from the base station during the $i$-th symbol interval can be written as $p[i] = S A b[i]$, where $S = [s_1, s_2, ..., s_K]$ is the matrix of spreading waveforms; and $A = \text{diag}(A_1, ..., A_K)$ contains the user signal amplitudes. The vector $p[i]$ is passed through a parallel-to-serial converter and transmitted over the multipath channel. The path delays are assumed to be an integral number of chip periods. Denote the multipath channel seen
by the $k$-th user as $f_k = [f_{k,1}, f_{k,2}, \ldots, f_{k,L}]^T$, where $L$ is the number of resolvable paths and $f_{k,l}$ is the complex fading gain corresponding to the $l$-th path of the $k$-th user. We assume that $L < N$. At the $k$-th user’s receiver, the $N \times 1$ received signal during $N$ consecutive chip intervals corresponding to $b[i]$ is given by

$$r_k[i] = F_k S A b[i] + n_k[i] \text{ with } F_k = \begin{bmatrix} f_{k,1} & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ f_{k,L} & \ddots & f_{k,1} & \ddots & \vdots \\ 0 & \ddots & \ddots & 0 \\ 0 & \cdots & f_{k,L} & \cdots & f_{k,1} \end{bmatrix}_{N \times N},$$

where $r_k[i] = [r_{k,1}[i], \ldots, r_{k,N}[i]]^T$ is the received signal, $n_k[i] \sim \mathcal{N}_c(0, \sigma^2 I_N)$ is the complex white Gaussian noise vector at the $k$-th receiver, and $H_k = F_k S$. Notice that we have assumed that ISI can be ignored either by being truncated or by inserting a guard interval.

At the $k$-th receiver, a linear detector to recuperate the signal $b_k[i]$ can be represented by an $N$-dimensional vector $w_k \in \mathbb{C}^N$, which is correlated with the received signal $r_k[i]$ in (1) to obtain $z_k[i] = w_k^H r_k[i]$, and the $k$-th mobile unit makes a decision $\hat{b}_k[i] = Q(z_k[i])$, where $Q$ rounds to the closest point in the constellation.

**Linear Decorrelating Detector:** The decorrelating detector completely eliminates the multiuser interference (MUI) and interchip interference (ICI), at the expense of enhancing the noise. The linear decorrelating detector for user $k$ is given by [12]

$$w_k = H_k^H e_k = H_k (H_k^H H_k)^{-1} e_k,$$

where $e_k$ denotes a $K$-dimensional vector with all entries zeros, except for the $k$-th entry, which is 1. The output of this detector is given by

$$z_k[i] = w_k^H r_k[i] = A_k b_k[i] + w_k^H n_k[i] \implies \text{SINR}_k = \frac{A_k^2}{\sigma^2 \|w_k\|^2},$$

where SINR$_k$ is the signal-to-interference-plus-noise ratio for the $k$-th user. Suppose that the QoS requirement for user $k$ is such that SINR$_k \geq \gamma_k$, where $\gamma_k$ is the minimum acceptable SINR value for user $k$. Hence we have $A_k^2 = \sigma^2 \gamma_k \|w_k\|^2$. And the total required transmit power is given by

$$P_T = \sum_{k=1}^{K} A_k^2 = \sum_{k=1}^{K} \sigma^2 \gamma_k \|e_k\|^2 (S^H F_k^H F_k S)^{-H} e_k.$$
**Linear MMSE Detector:** The linear MMSE detector for user \( k \) is given by [12]

\[
w_k = \arg \min_{w_k \in \mathbb{C}^N} \mathbb{E} \{ |b_k[i] - w_k^H r_k[i]|^2 \} = A_k (H_k^2 H_k^H + \sigma^2 I_N)^{-1} H_k e_k.
\] (5)

The SINR for this detector is given by

\[
\text{SINR}_k = \frac{A_k^2 \| w_k^H H_k e_k \|^2}{\sum_{j \neq k} A_j^2 \| w_k^H H_k e_j \|^2 + \sigma^2 \| w_k \|^2}.
\] (6)

We seek to minimize the total power \( P_T \) such that \( \text{SINR}_k \geq \gamma_k \). The iterative power control algorithm for linear MMSE MUD proposed in [11] can be extended to the downlink scenario. At the \((n+1)\)-th iteration, the MMSE filter \( w_k(n+1) \) is constructed using the current power matrix \( A(n) \). Then, the power matrix \( A(n+1) \) is updated using the new filter coefficients \( w_k(n+1) \).

Algorithm 1: Power control algorithm for linear MMSE MUD in the downlink

<table>
<thead>
<tr>
<th>INPUT: ( H_k, \gamma_k, \sigma^2 ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOR ( n = 0, 1, 2, \ldots ) DO</td>
</tr>
<tr>
<td>FOR ( k = 1, 2, \ldots, K ) DO</td>
</tr>
<tr>
<td>( w_k(n+1) = (H_k^2 A_k^2(n) H_k^H + \sigma^2 I)^{-1} A_k(n) H_k e_k )</td>
</tr>
<tr>
<td>( A_k^2(n+1) = \gamma_k \frac{\sum_{j=1}^K A_j^2(n) | w_k^H (n+1) H_k e_j |^2 + \sigma^2 (w_k^H (n+1) w_k(n+1))}{| w_k^H (n+1) H_k e_k |^2} ) (7)</td>
</tr>
<tr>
<td>END FOR;</td>
</tr>
<tr>
<td>END FOR;</td>
</tr>
<tr>
<td>OUTPUT: assigned powers ( A_k ) and linear MMSE filters ( w_k, k = 1, \ldots, K ).</td>
</tr>
</tbody>
</table>

### 3 Linear Precoding Schemes

In this section we consider different approaches to implement linear precoding assuming that the transmitter has perfect CSI.

#### 3.1 Bit-wise Linear Precoding

We assume that each mobile unit employs only a filter matched to its own spreading sequence, and it does not need to know other users’ spreading sequences or to estimate the channel.
Denote the symbol by symbol bit-wise precoding operation as \( \mathbf{x}[i] = \mathbf{M}_b \mathbf{A} \mathbf{b}[i] \), where \( \mathbf{x}[i] \) is the precoded symbol vector and \( \mathbf{M}_b \in \mathbb{C}^{K \times K} \) is the bit-wise linear precoding matrix. Then, after spreading the precoded data, the signal transmitted from the base station during the \( i \)-th symbol interval can be written as \( \mathbf{p}[i] = \mathbf{S} \mathbf{x}[i] = \mathbf{S} \mathbf{M}_b \mathbf{A} \mathbf{b}[i] \). The vector \( \mathbf{p}[i] \) is passed through a parallel-to-serial converter and transmitted through the channel. The signal received by the \( k \)-th user is then given by

\[
\mathbf{r}_k[i] = \mathbf{F}_k \mathbf{S} \mathbf{M}_b \mathbf{A} \mathbf{b}[i] + \mathbf{n}_k[i],
\]

where \( \mathbf{F}_k \) is given in (1). Then the corresponding matched filter \( \mathbf{s}_k \) is applied to \( \mathbf{r}_k[i] \). Stacking the outputs of the \( K \) matched-filters we obtain

\[
\begin{bmatrix}
\mathbf{s}_1^H \mathbf{r}_1[i] \\
\mathbf{s}_2^H \mathbf{r}_2[i] \\
\vdots \\
\mathbf{s}_K^H \mathbf{r}_K[i]
\end{bmatrix}
= \begin{bmatrix}
\mathbf{s}_1^H \mathbf{F}_1 \mathbf{S} \\
\mathbf{s}_2^H \mathbf{F}_2 \mathbf{S} \\
\vdots \\
\mathbf{s}_K^H \mathbf{F}_K \mathbf{S}
\end{bmatrix}
\mathbf{M}_b \mathbf{A} \mathbf{b}[i] +
\begin{bmatrix}
\mathbf{s}_1^H \mathbf{n}_1[i] \\
\mathbf{s}_2^H \mathbf{n}_2[i] \\
\vdots \\
\mathbf{s}_K^H \mathbf{n}_K[i]
\end{bmatrix}.
\]

The \( k \)-th receiver makes a decision \( \hat{\mathbf{b}}_k[i] = \mathcal{Q}(y_k[i]) \). Therefore the precoder design problem involves designing the precoding matrix \( \mathbf{M}_b \) such that \( \mathbf{r}[i] \) is as close to \( \mathbf{b}[i] \) as possible.

**Bit-wise Linear MMSE Precoder:** Assuming that the spreading sequences are normalized, the linear MMSE precoder chooses the precoding matrix \( \mathbf{M}_b \) to minimize \( \mathbb{E}\{\|\mathbf{b} - \mathbf{y}\|^2\} \), and is given by \[13\] \( \mathbf{M}_b = \beta \mathbf{H}_b^{-1}, \) with \( \beta = \sqrt{\frac{P_T}{\text{tr}(\mathbf{S} \mathbf{H}_b^{-1} \mathbf{A} \mathbf{H}_b^{-1} \mathbf{S}^H)}} \). Note that such a linear MMSE precoder also zero-forces the interference. If the constraint is the minimum SINR at each receiver \( \gamma_k \), we obtain the unconstrained precoding solution \( \mathbf{M}_b = \mathbf{H}_b^{-1}. \) Thus we have \( \text{SINR}_k = \frac{A_k^2}{\sigma^4} \); and the power assigned to the \( k \)-th user becomes \( A_k^2 = \sigma^2 \gamma_k. \) Then the total power required at the transmitter becomes \( P_T = \mathbb{E}\{\|\mathbf{S} \mathbf{M}_b \mathbf{A} \mathbf{b}[i]\|^2\} = \text{tr}(\mathbf{S} \mathbf{M}_b \mathbf{A}^2 \mathbf{M}_b^H \mathbf{S}^H). \)

**Bit-wise Wiener Precoder:** The bit-wise Wiener precoder is proposed in \([5, 6]\) as the matrix \( \mathbf{M}_b \) and constant \( \beta \) that minimize \( \mathbb{E}\{\|\mathbf{b}[i] - \beta^{-1} \mathbf{y}[i]\|^2\} \), subject to \( \mathbb{E}\{\|\mathbf{M}_b \mathbf{A} \mathbf{b}[i]\|^2\} = P_T. \) Given the total transmit power \( P_T \), the Wiener precoder is given by

\[
\mathbf{M}_b = \beta \mathbf{F}^{-1} \mathbf{H}_b^H, \quad \text{with} \quad \beta = \sqrt{\frac{P_T}{\text{tr}(\mathbf{F}^{-2} \mathbf{H}_b^H \mathbf{A}^2 \mathbf{H}_b)}} \quad \text{and} \quad \mathbf{F} = \mathbf{H}_b^H \mathbf{H}_b + \frac{K \sigma^2}{P_T} \mathbf{I}_N.
\]

**Optimal Transmit Spreading Sequences:** Besides optimizing the precoding matrix \( \mathbf{M}_b \) for a given channel realization, we can also optimize the transmit spreading sequences.
Denote \( s_1, \ldots, s_K \) as the fixed spreading sequences used at the mobile units (i.e., the matched filters) and \( \tilde{s}_1, \ldots, \tilde{s}_K \) as the optimized spreading sequences used at the transmitter. Denote \( \tilde{S} = [\tilde{s}_1, \ldots, \tilde{s}_K] \). Similarly to (9), the received signal can be written as

\[
\begin{bmatrix}
 s_1^H r_1[i] \\
 s_2^H r_2[i] \\
 \vdots \\
 s_K^H r_K[i]
\end{bmatrix}
= \begin{bmatrix}
 s_1^H F_1 \\
 s_2^H F_2 \\
 \vdots \\
 s_K^H F_K
\end{bmatrix} \tilde{S} M_b A b[i] + \begin{bmatrix}
 s_1^H n_1[i] \\
 s_2^H n_2[i] \\
 \vdots \\
 s_K^H n_K[i]
\end{bmatrix} v[i].
\] (11)

Following [10], it can be easily shown that the linear MMSE precoding matrix is given by \( M_b = (H_c \tilde{S})^{-1} \), and \( A_k^2 = \sigma^2 \gamma_k, k = 1, \ldots, K \). Next we show that for any given propagation channel \( F_1, \ldots, F_K \), original spreading sequences \( S \), and minimum SINR requirements, we can explicitly find the optimal spreading matrix \( \tilde{S} \in \mathbb{C}^{N \times K} \) such that the total transmit power \( P_T \) is minimized. Assume that the \( K \times N \) matrix \( H_c \) has rank \( K \), where \( N \geq K \).

Define the SVD \( H_c = U_c \Sigma_c V_c^H \), where \( U_c \) is a \( K \times K \) unitary matrix, \( V_c^H \) is an \( N \times N \) unitary matrix and \( \Sigma_c \) is a \( K \times N \) diagonal matrix with \([\Sigma_c]_{i,i} = \lambda_{c,i}\) being the positive square root of the \( i \)-th eigenvalue of \( H_c H_c^H \).

**Proposition 1** Given the channels \( F_1, \ldots, F_K \), the receiver matched-filters \( s_1, \ldots, s_K \), and the target SINR \( \gamma_1, \ldots, \gamma_K \) of all users, by optimizing the transmit spreading matrix \( \tilde{S} \) used in the bit-wise linear MMSE precoder, the minimum achievable transmit power is given by

\[
P_T^* = \min_{\tilde{S} \in \mathbb{C}^{N \times K}} \text{tr}(\tilde{S} M_b A^2 M_b^H \tilde{S}^H) = \sum_{k=1}^K A_k^2 \lambda_{c,k}^{-2},
\] (12)

where \( A_k^2 = \sigma^2 \gamma_k, k = 1, \ldots, K \) are the assigned powers. One solution to the optimization problem in (12) (i.e., the optimal transmit spreading matrix) is given by the \( N \times K \) matrix \( \tilde{S}^* = H_c^H \).

**Proof:** Note that \( M_b = (H_c \tilde{S})^{-1} \) and therefore the transmitted vector is given by \( p[i] = \tilde{S} M_b A b[i] = \tilde{S} (H_c \tilde{S})^{-1} A b[i] \). Denote the SVDs of \( H_c \) and \( \tilde{S} \) by \( H_c = U_c \Sigma_c V_c^H \) and...
$\tilde{S} = U_{\tilde{s}} \Sigma_{\tilde{s}} V_{\tilde{s}}^H$, respectively. Then the total transmit power is

$$P_T = \mathbb{E}\{p^H [i] p[i]\} = \text{tr}(S (H_c S)^{-1} A^2 (H_c S)^{-H} S^H)$$

$$= \text{tr}(U_{\tilde{s}} \Sigma_{\tilde{s}} V_{\tilde{s}}^H (U_c \Sigma_c V_c^H U_{\tilde{s}} \Sigma_{\tilde{s}} V_{\tilde{s}}^H)^{-1} A^2 (U_c \Sigma_c V_c^H U_{\tilde{s}} \Sigma_{\tilde{s}} V_{\tilde{s}}^H)^{-H} V_{\tilde{s}} \Sigma_{\tilde{s}}^H U_{\tilde{s}}^H)$$

$$= \text{tr}(\Sigma_{\tilde{s}} V_{\tilde{s}}^H V_{\tilde{s}} T^{-1} U_c^H A^2 U_c T^{-H} V_{\tilde{s}}^H V_{\tilde{s}} \Sigma_{\tilde{s}}^H)$$

$$= \text{tr}(\Sigma_{\tilde{s}} T^{-1} A^2 T^{-H} \Sigma_{\tilde{s}}^H) = \text{tr}(A^2 \Sigma_{\tilde{s}}^2 T^{-1} T^{-H}), \quad (13)$$

where $T = \Sigma_c V_c^H U_s \Sigma_s$ is a $K \times K$ matrix; $\Sigma_{\tilde{s}}^2 = \Sigma_{\tilde{s}}^H \Sigma_{\tilde{s}}$ is a $K \times K$ diagonal matrix; and we used the fact that $U_s, U_c, V_c$ and $V_s$ are unitary.

Consider $T$ expressed in terms of the matrices obtained with the thin SVD [4], $T = \Sigma_{\tilde{s}}^{(t)} C \Sigma_{\tilde{s}}^{(t)}$, where $\Sigma_{\tilde{s}}^{(t)}$ and $\Sigma_{\tilde{s}}^{(t)}$ are the $K$-th leading submatrix of $\Sigma_{t}$ and $\Sigma_{\tilde{s}}$, respectively; and $C = V_{\tilde{s}}^t U_{\tilde{s}}^{(t)}$ is a $K \times K$ matrix (where $V_{\tilde{s}}^{(t)}$ and $U_{\tilde{s}}^{(t)}$ denote the matrices consisting of the first $K$ columns of $U_{\tilde{s}}$ and $V_{\tilde{s}}$, respectively). Denoting $\{v_{c,1}, ..., v_{c,K}\}$ and $\{u_{\tilde{s},1}, ..., u_{\tilde{s},K}\}$ as the first $K$ columns of $V_c$ and $U_{\tilde{s}}$, respectively, we have $[C]_{ij} = \langle v_{c,i}, u_{\tilde{s},j}\rangle, i = 1, ..., K$. Next we show that the eigenvalues of $C$ denoted as $\phi_i$, $i = 1, ..., K$, always satisfy $|\phi_i| \leq 1$.

Denote $\{e_1, ..., e_K\}$ as the orthogonal basis of the $K$-dimensional space. Then the $l$-th component of the $C$ transform of the $j$-th basis is given by $[e_j^t]_l = [Ce_j]_l = [C]_{l,j} = \langle v_{c,l}, u_{\tilde{s},j}\rangle$, where $\langle \cdot, \cdot \rangle$ denotes the inner product. Hence $\|e_j^t\|^2 = \sum_{l=1}^K |\langle v_{c,l}, u_{\tilde{s},j}\rangle|^2$. Notice that since $V_c$ and $U_{\tilde{s}} \in \text{SU}(N)$ (i.e., special unitary group), then $V_{\tilde{s}}^t U_{\tilde{s}}$ also belongs to the SU($N$); and therefore the $L_2$ norm of each column vector of the $N \times N$ matrix $V_{\tilde{s}}^t U_{\tilde{s}}$ equals to one, i.e., $\sum_{l=1}^N |\langle v_{c,l}, u_{\tilde{s},j}\rangle|^2 = 1, j = 1, ..., N$. Since $N \geq K$, we have $\|e_j^t\|^2 = \sum_{l=1}^K |\langle v_{c,l}, u_{\tilde{s},j}\rangle|^2 \leq 1, j = 1, ..., K$. This is, the $L_2$ norm of the transformation by $C$ of every basis vector is always less or equal to 1. Every vector in the $K$-dimensional space can be written as a linear combination of the basis and therefore, the $C$ transform applied to any vector reduces the norm. In particular, it reduces the norm of the eigenvectors of $C$. Therefore, we conclude that the eigenvalues of $C$ satisfy $|\phi_i| \leq 1, \forall i$.

Substituting $T^{-1} = [\Sigma_{\tilde{s}}^{(t)}]^{-1} C^{-1} [\Sigma_{\tilde{s}}^{(t)}]^{-1}$ and the eigenvalue decomposition of $C = W \Phi W^{-1}$ (where $\Phi = \text{diag}(\phi_1, ..., \phi_K)$) in (13) we obtain

$$P_T = \text{tr}(A^2 \Sigma_{\tilde{s}}^2 T^{-1} T^{-H}) = \text{tr}(A^2 C^{-1} C^{-H} \Sigma_{\tilde{s}}^{-2}) = \text{tr}(A^2 \Phi^{-1} \Phi^{-H} \Sigma_{\tilde{s}}^{-2})$$

$$= \sum_{i=1}^K A_i^2 \lambda_{c,i}^{-2} |\phi_i|^{-2} \geq \sum_{i=1}^K A_i^2 \lambda_{c,i}^{-2}. \quad (14)$$

Denote the thin SVD of $\tilde{S} = U_{\tilde{s}}^{(t)} \Sigma_{\tilde{s}}^{(t)} V_{\tilde{s}}^{(t)}$. Finally, with $\tilde{S}^*= H_c^H$, the thin SVD decomposition becomes $\tilde{S}^* = (U_c^{(t)} \Sigma_c^{(t)} V_c^{(t)H})^H = V_c^{(t)} \Sigma_c^{(t)H} U_c^{(t)H}$, i.e., $U_{\tilde{s}}^{(t)} = V_c^{(t)}$. Therefore
\[ C = V_c^{(t)H} U_s^{(t)} = I_K, \] and \( C \) has unit eigenvalues. Hence we have equality in (14) and \( \tilde{S}^* = H_c^{H} \) is an optimal spreading matrix for linear MMSE precoding.

Remark: There are many other forms of the optimal spreading matrix \( \tilde{S}^* \) such that \( C = V_c^{(t)H} U_s^{(t)} \) has unit eigenvalues. Specifically, we need to construct an \( N \times N \) matrix \( U_\tilde{s} \) that rotates the first \( K \) columns vectors of \( V_c \) in the same \( K \)-dimensional subspace and keep invariant the \( N-K \)-dimensional subspace spanned by the \( N-K \) remaining vectors. Consider first the real case. The constraints on the \( K \) first columns of \( U_\tilde{s} \) are: (a) \( \sum_{i=1}^{K} |\langle v_c,i, u_\tilde{s},j \rangle|^2 = 1, j = 1, ..., K. \) [\( K \) equations.] (b) \( \langle u_\tilde{s},i, v_c,m \rangle = 0, i = 1, ..., K; m = K+1, ..., N. \) [\( K \cdot (N-K) \) equations.] (c) \( \langle u_\tilde{s},i, u_\tilde{s},j \rangle = \delta_{ij}, i, j = 1, ..., K. \) \( [(K-1)+(K-2)+...+(K-K+1)+(K-K) = K^2-\frac{1}{2}K(K+1) \) equations.] To construct \( U_\tilde{s} \), there are \( NK \) variables in the \( K \) first columns of \( U_\tilde{s} \). After subtracting the number of constraints, we have \( (K^2-K)/2 \) degrees of freedom, which is nothing more than the dimension of the \( O(K) \) (i.e., orthogonal group) as expected.

In the complex case, there are \( 2NK \) variables in the first \( K \) columns of \( U_\tilde{s} \) and it can be shown that the solution generalizes to \( (K^2-1) \) degrees of freedom that is the number of free parameters of the \( SU(K) \). To summarize, to construct the optimal spreading matrix with SVD decomposition \( \tilde{S} = U_\tilde{s} \Sigma_\tilde{s} V_\tilde{s}^{H} \), we only have to find the unitary matrix \( U_\tilde{s} \) satisfying the above constraints on its \( K \) first column vectors (i.e., range of \( \tilde{S} \)). Moreover, there are \( (K^2-1) \) degrees of freedom to select it.

### 3.2 Chip-wise Linear Precoding

In chip-wise precoding, we do not explicitly use any spreading matrix at the transmitter. This is, the precoder takes \( K \) symbols and outputs the spread vector of length \( N \). Hence the spreading and precoding operations are effectively combined. The received signal at the \( k \)th receiver is given by

\[
r_k[i] = F_k M_c b[i] + n_k[i],
\]

where \( M_c \in \mathbb{C}^{N \times K} \) is the chip-wise precoding matrix. At each receiver \( k \), the matched-filter \( s_k \) is applied to \( r_k[i] \). By stacking the outputs of all \( K \) matched-filters we obtain

\[
\begin{bmatrix}
    s_1^H r_1[i] \\
    s_2^H r_2[i] \\
    \vdots \\
    s_K^H r_K[i]
\end{bmatrix}
= 
\begin{bmatrix}
    s_1^H F_1 \\
    s_2^H F_2 \\
    \vdots \\
    s_K^H F_K
\end{bmatrix}
M_c A b[i] + 
\begin{bmatrix}
    s_1^H n_1[i] \\
    s_2^H n_2[i] \\
    \vdots \\
    s_K^H n_K[i]
\end{bmatrix}
\begin{bmatrix}
    y[i] \\
    H_c 
\end{bmatrix}.
\]

(16)
Differently from the bit-wise system model, here the channel matrix $H_c$ is not a square matrix but has dimension $K \times N$ with $N \geq K$.

**Chip-wise MMSE Precoding:** Using an argument similar to [10], the linear MMSE chip-wise precoder is given by

$$M_c = H_c^\dagger = H_c^H (H_c H_c^H)^{-1}. \quad (17)$$

It is easily seen that the SINR for each user is given by

$$\text{SINR}_k = \frac{A_k^2}{\sigma^2}, \quad k = 1, ..., K. \quad (18)$$

As before, if we assume that the required SINR for user $k$ is $\gamma_k$, the required power assigned to the $k$-th user becomes $A_k^2 = \sigma^2 \gamma_k$. Due to the precoding matrix, the required total transmit power becomes

$$P_T = \text{tr}(H_c^\dagger A^2 H_c^H) = \text{tr}(A^2 (H_c H_c^H)^{-1}). \quad (19)$$

**Remark:** Note that under a fixed transmit power budget $P_T$, the linear MMSE precoder is given by $M_c = \beta H_c^\dagger$ with $\beta = \sqrt{P_T / \text{tr}(A^2 (H_c H_c^H)^{-1})}$ and $\text{SINR}_k = \frac{(\beta A_k)^2}{\sigma^2}$.

**Proposition 2** The above chip-wise linear MMSE precoding method is equivalent to the bit-wise linear MMSE precoding method with the optimal spreading matrix at the transmitter $\tilde{S}^*$. 

**Proof:** Using the SVD of $H_c = U_c \Sigma_c V_c^H$, the total transmit power required in the linear MMSE chip-wise precoder is given by

$$P_T = \text{tr}(H_c^\dagger A^2 H_c^H) = \text{tr}(V_c \Sigma_c^{-1} U_c^H A^2 U_c \Sigma_c^{-1} V_c^H)$$

$$= \text{tr} \left( A^2 \Sigma_c^{-2} \right) = \sum_{i=1}^{K} A_i^2 \lambda_{c,i}^{-2}. \quad (20)$$

Hence the transmit power with the chip-wise linear MMSE precoder is equal to the minimum transmit power in the bit-wise solution given in (12).

**Remark:** The above result shows that it is not necessary to optimize the spreading operation at the transmitter. That is, by applying the simple chip-wise precoding operation we can obtain the optimal performance.

**Chip-wise Wiener Precoding:** The Wiener precoder given in (10) can be used in our chip-wise scheme by substituting $H_b$ by $H_c$, resulting in the precoding matrix $M_c \in \mathbb{C}^{N \times K}$. 

9
Next we propose a power loading algorithm that can be applied to both the bit-wise and chip-wise Wiener precoders. Consider the signal model (16). Define $G = H_c M_c$. Then we can write

$$y_k[i] = A_k G_{kk} b_k[i] + \sum_{i=1,i\neq k}^K A_i G_{ki} b_i[i] + v_k[i], k = 1, ..., K.$$ 

In the Wiener precoder $M_c$ is not the pseudo-inverse of $H_c$ and therefore $G$ is not a diagonal matrix. Hence, for a fixed loading matrix $A$, the received SINR is given by

$$\text{SINR}_k = \frac{A_k^2 \|G_{kk}\|^2}{\sigma^2 + \sum_{i=1,i\neq k}^K A_i^2 \|G_{ki}\|^2}.$$ 

To achieve the target SINR $\gamma_k$ for each user $k$, we need to find the optimal powers $A_k^2, k = 1, ..., K$. Now, different from the linear MMSE precoding, the power allocation problem is coupled with the problem of finding the optimal precoding matrix. Following the ideas of \cite{11} we propose the following iterative algorithm to solve the joint problem. In the algorithm we first fix the power loading values $A(n)$ to find the precoding matrix and then, based on the precoding matrix, the power loading values are updated. Simulations show that the algorithm converges in about two or three iterations.

**Algorithm 2** Power control algorithm for Wiener precoder

**INPUT:** $H_c, \sigma^2$ and $\gamma_k, k = 1, ..., K$;

FOR $n = 1, 2, ...$ DO

$$F(n+1) = H_c^H H_c + \frac{K \sigma^2}{P_T(n)} I_N$$

$$\beta(n+1) = \sqrt{\text{tr} \left( F^{-2}(n+1) H_c^H A^2(n) H_c \right)}$$

$$M_c(n+1) = \beta(n+1) F^{-1}(n+1) H_c^H ;$$

$$G(n+1) = H_c M_c(n+1) ;$$

FOR $k = 1 : K$ DO

$$A_k^2(n+1) = \gamma_k \frac{\sum_{i=1,i\neq k}^K A_i^2(n) \|G_{ki}(n+1)\|^2 + \sigma^2 \|G_{kk}(n+1)\|^2}{\|G_{kk}(n+1)\|^2} ;$$

END;

$$P_T(n+1) = E\{\|M_c(n+1) A(n+1) b\|^2\} = \text{tr} \left( M_c(n+1) A^2(n+1) M_c^H(n+1) \right) ;$$

END FOR;

**OUTPUT:** precoding matrix $M_c(n+1)$, and assigned powers $A(n+1)$

4 Simulation Results

**Chip-wise precoding vs. bit-wise precoding:** We first compare the bit-wise linear MMSE precoding (without optimizing the spreading sequences at the transmitter) with the
chip-wise linear MMSE precoding. We assume that the target SINR per user is constant for all users, $\gamma_k = 10$dB, $k = 1, \ldots, K$. We consider random codes and Gold codes with spreading gain $N = 31$ and the total number of users $K = 15$. We assume that each mobile user experiences an independent multipath channel $f_k = [f_{k,1}, \ldots, f_{k,L}]^T$ with $L = 3$ resolvable paths, and the transmitter has perfect CSI of all users. The path gains are generated according to $f_{k,i} \sim \mathcal{N}_c(0, \frac{1}{L})$. The results are averaged over 1000 different channel realizations.

The cumulative distribution function (CDF) of the required power at the transmitter to achieve the minimum SINR at the receivers is shown in Fig. 1. It is seen that under this severe multipath, the suboptimal bit-wise solution incurs a large performance degradation.

**Chip-wise precoding with matched-filter vs. bit-wise precoding with RAKE receiver:** The bit-wise linear MMSE precoding with a RAKE receiver was proposed in [13]. The difference with the linear MMSE precoder considered in the Section 3.1 is that the receiver must also estimate the channel and apply a RAKE receiver, consequently, increasing the number of pilot symbols and the complexity of the receiver. We discuss this method only for comparison since we seek precoding solutions with simple receivers with no receiver CSI.

The RAKE receiver can be implemented with a matched filter using the effective spreading sequence (i.e., the $k$-th effective spreading sequence is $\bar{s}_k = f_k \ast s_k$) instead of the original spreading sequence. With our notation, the $k$-th effective spreading sequence is given by the convolution $\bar{s}_k = F_k Se_k = F_k s_k$, where we have limited the convolution to $N$ chip samples.

Then, with the RAKE receiver the system model can be written as

$$
\begin{bmatrix}
    s_1^H F_1^H r_1[i] \\
    s_2^H F_2^H r_2[i] \\
    \vdots \\
    s_K^H F_K^H r_K[i]
\end{bmatrix}
\begin{bmatrix}
    s_1^H F_1^H F_1 S \\
    s_2^H F_2^H F_2 S \\
    \vdots \\
    s_K^H F_K^H F_K S
\end{bmatrix}
M_b Ab[i] +
\begin{bmatrix}
    s_1^H F_1^H n_1[i] \\
    s_2^H F_2^H n_2[i] \\
    \vdots \\
    s_K^H F_K^H n_K[i]
\end{bmatrix}

\begin{bmatrix}
    y[i] \\
    H_b \\
    v[i]
\end{bmatrix}.
$$

(22)

It is easily seen that the linear MMSE precoding solution still yields $M_b = H_b^{-1}$, where $H_b$ is defined in (22). The signal to noise ratio for user $k$ is

$$
\text{SINR}_k = \frac{A_k^2}{\sigma^2 \| F_k s_k \|^2}, \quad k = 1, \ldots, K,
$$

(23)

and the required power to achieve an SINR value $\gamma_k$ becomes $A_k^2 = \sigma^2 \gamma_k s_k^H F_k^H F_k s_k$. Therefore, the total transmitted power is given by

$$
P_T = \mathbb{E}\{\| S H_b^{-1} Ab[i] \|^2 \} = \text{tr}(S H_b^{-1} A^2 H_b^{-H} S^H).
$$

(24)
Notice that the Wiener precoding solution can also be applied to the system in (22).

Next we compare the chip-wise linear MMSE precoder given in Section 3.2 (which is equivalent to the optimal bit-wise linear MMSE precoder) with the above bit-wise precoder with a RAKE receiver. The results are shown in Fig. 2. With Gold sequences the RAKE receiver brings less than 0.5dB gain on average compared with the simple chip-wise precoder with matched-filter receiver. Note that the performance of a communication system is dominated by the outage events. Given an outage probability \( p_{\text{out}} \), we define the corresponding outage power \( P_{\text{out}} \) as \( p_{\text{out}} = Pr\{P_T \geq P_{\text{out}}\} \). It is seen that although on average the RAKE receiver is slightly better, it is more prone to outage. For instance, consider in the plot the 5% outage probability for which the chip-wise precoder requires around 34.5 dB whereas the RAKE receiver requires around 35.5dB. When considering the 1% outage probability, this effect is more pronounced and the RAKE receiver requires 5 dB more than the chip-wise precoder to achieve the same performance. This effect will be more clear in the BER simulation results [cf. Fig.5 and Fig.6]. Interestingly, the performance of the precoder with RAKE receiver decays considerably when random sequences are used. Therefore, the chip-wise precoder is not only simpler (and it makes the receiver simpler since no CSI is required at the receiver) but it also has excellent performance. From the above simulation results we can conclude that: (a) The original bit-wise precoder with the matched filter at the receiver is far from optimal in multipath channels; (b) The bit-wise precoder with RAKE receiver makes the mobile units more complex and does not bring much improvements with Gold sequences and it can be very detrimental with random spreading sequences; (c) Therefore the proposed chip-wise precoding method offers both low complexity and high performance.

**Linear precoding vs. linear MUD – total transmit power:** Next we compare linear MUD with linear precoding assuming the same simulations parameters. We compare the CDF of the required total power \( P_T \) at the transmitter to achieve a target SINR \( \gamma_k = 13\text{dB} \), \( \forall k \), in each of the four following schemes: (a) linear decorrelating MUD [cf. Eq.(4)]; (b) linear MMSE MUD [cf. Alg. 1]; (c) chip-wise linear MMSE precoder, [cf. Eq.(19)]; and (d) chip-wise Wiener precoder [cf. Alg. 2]. Simulations are performed for spreading gain \( N = 31 \), with Gold and random spreading sequences. Fig. 3 shows the results with \( K = 15 \) users and Fig. 4 shows the results with \( K = 27 \) users. It is seen that with Gold codes, MUD is slightly better (although only 0.5dB of difference with linear precoding when 15 users are considered), whereas with random codes linear precoding largely outperforms MUD. Notice that the Wiener precoder is slightly better than the MMSE precoder. It is also seen that
the total power required in the precoding solutions is almost independent of the chosen spreading sequences and therefore, an outage event is less likely to occur. Although the linear MMSE MUD solution seems to be quite effective with Gold codes, we recall that it is unlikely to be implemented in the downlinks of most wireless systems due to the amount of required feedback information to implement perfect power control and other issues discussed in Section 1. Also notice that the linear decorrelator offers very poor performance in heavily loaded systems, which does not occur to the linear MMSE linear precoder.

**Linear precoding vs. linear MUD – BER performance:** Fig. 5 and Fig. 6 show the BER performance of the various linear MUD and linear precoding methods. The results are averaged over 100 channel realization and QPSK modulation is employed. Recall that the linear MMSE precoder is equivalent to the transmitter counterpart of the decorrelator. For the decorrelating MUD we consider perfect power loading to achieve the same SNR across the users. It is seen that the linear MMSE precoder with RAKE only performs slightly better with Gold sequences in the very low SNR region. In all the other cases, the chip-wise linear MMSE precoder obtains much better results. On the other hand, the chip-wise MMSE precoder obtains much better results than the decorrelating MUD, especially in heavily loaded systems. These results are due to the outage events of the decorrelating MUD observed in Fig. 3 and Fig. 4. Again, it is seen that the BER performance of the chip-wise precoding solution is almost independent of the chosen spreading sequence.

### 5 Downlink User Scheduling for Linear Precoding

Scheduling is a technique to increase the utilization of the wireless medium. For example, in the recently proposed multiuser opportunistic scheduling scheme [8] the schedulers opportunistically exploit channel variations of multiple users to select the best set of users to transmit data subject to fairness (e.g., maximum delay), QoS (e.g., minimum SNR), and resource constraints (e.g., maximum power available at the transmitter) [7], to obtain a significant increase of total system throughput. In general the number of users that can be simultaneously supported by the system is small and thus, there are a large number of possible user subset selections when the number of users in the system is large. Straightforward implementation of the user subset selection by simple exhaustive enumeration suffers from high computational complexity.

In this section, we propose user subset selection algorithms that can be naturally im-
plemented in precoded systems. We assume that the satisfaction that a user receives in a system (i.e., the utility) is a binary function that takes zero value when the SINR is below a threshold and takes unit value when the SINR is above the threshold. This is appropriate for voice or video-on-demand applications in which the SINR above a threshold will not provide additional benefit and the SINR below the threshold leads to unintelligible speech or video. As we have shown in the previous section, the linear MMSE and Wiener precoders achieve the similar performance. However, the Wiener precoder has a higher complexity because the power control has to be computed iteratively. Therefore in this section we restrict ourselves to the linear MMSE precoder. One important property of the chip-wise precoder proposed in Section 3.2 is that in the channel matrix $H_c$, each row depends only on the spreading sequence and channel of one particular user (actually, each row in $H_c$ is the effective spreading sequence, i.e., the convolution between the channel response and the spreading sequence of that particular user). Note that in the bit-wise solution this is not the case and each row depends on the spreading sequence of all the active users. This property will allow us to propose low-complexity algorithms for the chip-wise precoder.

5.1 Maximum User Allocation – Optimal Solution

Our objective is to accommodate as many users as possible such that if user $k$ is active, $\text{SINR}_k \geq \gamma$, assuming that the base station is constrained to a maximum power budget $P_T$. Recall that in the chip-wise solution, for a fixed power budget $P_T$, the SINR for the $k$-th user is given by

$$\text{SINR}_k = \frac{\beta^2 A_k^2}{\sigma_n^2}, \quad \text{with } \beta = \sqrt{\frac{P_T}{\text{tr}(A^2 (H_c H_c^H)^{-1})}}.$$  \hspace{1cm} (25)

Since a same target SINR value $\gamma$ is assumed for all users, the users should have the same transmit power and hence can assume $A_k = 1, \forall k$. Therefore, the QoS constraint $\text{SINR}_k \geq \gamma$ translates into the following condition on $H_c$:

$$\text{tr}((H_c H_c^H)^{-1}) \leq \frac{P_T}{\sigma^2 \gamma}. \hspace{1cm} (26)$$

Denote $U$ as the total number of users in the network, $\theta$ as the user subset selected, and $|\theta|$ as the number of users in $\theta$ (e.g., selecting the first and third users corresponds to $\theta = \{1, 3\}$ and $|\theta| = 2$). The channel matrix corresponding to the active users is $H_\theta$ where $H_\theta$ is the submatrix of $H_c$ (where $H_c$ has $U$ rows) obtained from the rows indicated in $\theta$. Let $\Theta$ be the set of all possible user subsets. Therefore, the total number of possible user
subsets is $|\Theta| = \sum_{k=0}^{U} \binom{U}{k}$. Denote $\Omega$ as the set of feasible user selections in $\Theta$, i.e.,

$$\Omega = \{ \theta \in \Theta : \text{SINR}_k \geq \gamma, \ \forall k \in \theta \} = \{ \theta \in \Theta : \text{tr}((H_\theta H_\theta^H)^{-1}) \leq P_T/(\sigma^2 \gamma) \}. \quad (27)$$

Then the optimization problem becomes finding $\theta \in \Omega$ such that $|\theta|$ is maximized. This is a highly complex combinatorial problem since for each possible solution in $\Theta$, a matrix pseudoinverse needs to be computed. Thus the total number of complex multiplications required is $\sum_{k=1}^{|\theta|+1} \binom{U}{k} (k^3 + Nk^2)$.

### 5.2 Low Complexity Algorithms

Next we propose low-complexity algorithms that employ a greedy approach to add or remove one user at a time. As mentioned before, in addition to being the optimal linear precoder, the advantage of using the chip-wise linear precoding method is that adding or removing one user corresponds to adding or removing a row to the channel matrix $H_c$ and the rest of the rows remain unchanged. Note that the performance only depends on the selected users and not on the order in which the users are selected. This is, for any reordering in rows of $H_c$, the required power is equivalent. Any reordering of the rows can be expressed as $H'_c = PH_c$ where $P$ is a permutation matrix and hence $P^{-1} = P^H$. Therefore, $\text{tr}((H'_c H'_c^H)^{-1}) = \text{tr}((PH_c H_c^H P^H)^{-1}) = \text{tr}((H_c H_c^H)^{-1})$.

**Maximum Frobenius Norm Criterion:** An intuitive and classical approach in user allocation is to activate the users that see the best propagation channel. Two approaches can be taken: incremental allocation and decremental allocation. In the incremental allocation algorithm, the base station starts without selecting any user. At each step of the algorithm, it selects the user with maximum channel gain (i.e., maximum norm of the corresponding row of the chip-wise matrix). Then, the algorithm checks if (26) holds. If it does, the corresponding user is allocated. This is repeated until no more users can be allocated, i.e., until (26) no longer holds, or $|\theta| = \min(U, N)$. On the other hand, the decremental algorithm starts by assuming that all $|\theta| = \min(U, N)$ users with best channels are active. And it removes one user at a time until (26) is satisfied. The removed user is the one with the worst channel quality, i.e., with the lowest channel gain. Obviously if the number of active users is expected to be small, it is better to use the incremental algorithm. The main disadvantage of the user allocation approaches described above is that for every new user added, the matrix inverse in (26) cannot be reused.
Geometrical Criterion - Incremental Selection: We have already mentioned that users with good channel qualities (i.e., large path gains) are in general good candidates to be allocated. However, due to the precoding operation, a matrix inverse needs to be computed. Therefore, users with very large path gains but with highly correlated effective signature sequences (i.e., rows in the matrix $H_c$ close to parallel) can have a very negative effect in the required power at the transmitter. Therefore here we propose to select users based not only on the gains but also on the correlations (i.e., angles) between the respective effective signature sequences.

Assume that $K = |\theta|$ users have already been allocated, i.e., $H_\theta$ with rows $h_1, \ldots, h_K$. Then we propose to select a new row $h_i$ from the $(U - K)$ remaining ones (i.e., users not allocated yet) such that the projection onto the orthogonal complement of the already selected rows is maximum, i.e.,

$$\max_i \| \pi^+(h_i) \|, \quad i \in \{\text{non-selected users}\},$$

where $\pi^+(h_i)$ denotes the projection of $h_i$ on span$(h_1, \ldots, h_K)^\perp$ and $(\cdot)^\perp$ denotes the orthogonal complement. We consider a greedy incremental approach. The algorithm starts by selecting one row with the maximum norm and at every iteration the algorithm adds the row with the largest projection onto the orthogonal complement of the subspace spanned by the rows already selected. This selection can be implemented with the help of the Gram-Schmidt procedure. At every step of the algorithm, (26) needs to be checked to see if a new user can be allocated given the total power budget $P_T$. For every new user added, (26) requires a matrix inverse. Next, we propose a method to compute the matrix inverse recursively.

Denote the LQ decomposition of a $K \times N$ matrix as $H = LQ$ where $L$ is $K \times K$ lower left triangular and $Q$ has dimension $K \times N$ with $QQ^H = I_K$. The LQ decomposition can be obtained using Gram-Schmidt where the row vectors in $Q$, i.e., $q_1, \ldots, q_K$ are given by the recursion

$$q_1 = h_1/\|h_1\|, \quad \text{and} \quad q_i = \frac{h_i - \sum_{j=1}^{i-1} \mu_{ij} q_j}{\|h_i - \sum_{j=1}^{i-1} \mu_{ij} q_j\|}, \quad \text{for} \quad i = 2, \ldots, K,$$

where the Gram-Schmidt coefficients form the lower triangular matrix $L$ and are given by

$$\mu_{ij} = \langle h_i, q_j \rangle, \quad j < i, \quad \text{and} \quad \mu_{ii} = \|h_i - \sum_{j=1}^{i-1} \mu_{ij} q_j\|.$$  

By simple inspection, we have that $[L]_{ij} = \mu_{ij}$, and $\mu_{jj}$ is the value required in (28). Therefore, the LQ decomposition does not require any extra computations if we use the greedy geometrical user allocation.
Assume that one knows the LQ decomposition of $H$. Then, (26) can be evaluated using

$$\text{tr}((HH^H)^{-1}) = \text{tr}((LQQ^H L^H)^{-1}) = \|L^{-1}\|_F^2. \quad (31)$$

Note that (31) can be computed recursively as follows. Assume that we have computed $L^{-1}_{i-1}$ of size $(i-1) \times (i-1)$. Then, after selecting the new user (i.e., add one row to $H$), the $(i-1)$-th leading submatrix of $L^{-1}_i$ is given by $L^{-1}_{i-1}$ available from the previous iteration and the last row in $L^{-1}_i$ is given by

$$l^{-1}_i = \frac{1}{\mu_{i,i}}(e_i - \sum_{j=1}^{i-1} \mu_{ij}l^{-1}_j), \quad (32)$$

which follows from the Gauss-Jordan elimination and the relationship between the Gram-Schmidt coefficients and the triangular matrix $L$. Hence (31) is computed recursively as

$$\|L^{-1}_i\|_F^2 = \|L^{-1}_{i-1}\|_F^2 + \|l^{-1}_i\|_2^2. \quad (33)$$

Finally a low-complexity incremental selection algorithm for user allocation is summarized in Algorithm 3. Clearly, the complexity is dominated by the computation of all the Gram-Schmidt coefficients in step $\Diamond$ computed using (30), which requires $\sum_{i=1}^{\|\theta\|} N(U-i)i$ complex multiplications. The total complexity of the algorithm is upper bounded by $NU\|\theta\|^2$ complex multiplications.

### 5.3 Simulation Results

Next we give some simulation results to illustrate the performance of the different user allocation algorithms when the chip-wise linear precoder is employed.

We first consider the average number of users that each algorithm is able to allocate with respect to the total available power at the transmitter $P_T$. We set $\gamma = 12$dB. We assume that each mobile user experiences an independent multipath channel $f_k = [f_{k,1}, \ldots, f_{k,L}]^T$ with $L = 3$ resolvable paths and the transmitter has perfect CSI of all users. The path gains are generated according to $f_{k,i} \sim N_c(0, \frac{1}{L})$. We consider random spreading sequences with spreading gain $N = 8$, and $U = 12$ available users in the region. Fig. 7 illustrates the average number of users allocated, i.e., $\|\theta\|$ with respect to $P_T$ by different algorithms. It is seen that the low-complexity geometrical incremental algorithm achieves almost the optimal performance. Note that for instance, with $P_T = 26$dB, the optimal algorithm allocates around 7 users and it would require $\sum_{i=0}^{8} \binom{12}{i} (i^3 + Ni^2) = 1931664$ complex multiplications,
Algorithm 3 Low-complexity user allocation based on geometrical criterion

INPUT: all row vectors $h_1, ..., h_U$, $\gamma$, $P_T$, $\sigma$.

$\theta = \emptyset$; $P_r = 0$; % start without any user selected

FOR $i = 1, 2, \ldots$

FOR EVERY $j \in \{\{1, \ldots, U\} \setminus \theta\}$ DO % every user not selected yet

$b_j = h_j - \sum_{p=1}^{i-1} \mu_{j,p} q_p$

END FOR

$k_i = \arg\max_j \{b_j b_j^H\}$; % user with max projection onto orthogonal complement

$q_i = b_{k_i} / \|b_{k_i}\|$; % the new Gram-Schmidt vector

$l_i^{-1} = \frac{1}{\mu_{i,i}} (e_i - \sum_{t=1}^{i-1} \mu_{i,t} l_t^{-1})$; % last row in the new $L^{-1}$

$P_r = P_r + \sigma^2 \gamma \|l_i^{-1}\|^2$; % power required if we allocate this user $k_i$

IF $P_r < P_T$

$\theta = \theta \cup k_i$; % allocate this user and continue

IF $|\theta| = \min(U, N)$ THEN BREAK; % finish the algorithm

ELSE

$P_r = P_r - \sigma^2 \gamma \|l_i^{-1}\|^2$;

BREAK; % finish the algorithm

END IF

END FOR

OUTPUT: selected users $\theta$, required power $P_r$, selected submatrix $H_\theta$ and $H_\theta^H = Q^H L^{-1}$.

whereas the complexity for the proposed low-complexity algorithm is upper bounded by $NU|\theta|^2 = 6144$ complex multiplications. As the number of users increases, the optimal solution becomes intractable. It is seen that under this scenario, the maximum Frobenius norm selection criterion incurs a loss of between 2-4dB.

Next, to illustrate the effectiveness of the different algorithms, we consider a hypothetical scenario in which $K$ users need to be allocated. The $K$ users are chosen among the $U$ available users in the network using either optimal selection, maximum gain selection, or low-complexity geometrical selection. We look at the total power required at the transmitter $P_T$ to obtain $\gamma = 12$dB across the $K$ selected users. Fig. 8 shows the results with $U = 8$ available users, and Hadamard spreading sequences with $N = 8$. It is seen that the geometrical algorithm again achieves almost the optimal performance. Fig. 9 shows the results with $U = 16$ available users, spreading gain $N = 8$ and $K = 4$. It is seen that
as the number of possible combinations increases, the maximum Frobenius norm criterion incurs performance loss whereas the the geometrical algorithm is quite robust. Note that with $U = 16$ and $K = 4$, the optimal algorithm would compute $^{U\choose K}(NK^2+K^3)= 349440$ complex multiplications, whereas the low complexity algorithm would require less than $K^2NU = 2048$ complex multiplications.

6 Conclusions

In this work we have compared the performance of linear precoding and linear MUD in the downlink of TDD-CDMA systems. We have proposed different linear precoding schemes and our results reveal that in general precoding can outperform the more complex MUD. Moreover, we have shown that the proposed chip-wise linear MMSE precoding method is optimal in the sense that it requires the minimum total transmitted power to meet a certain receiver QoS performance. These results strongly motivate the use of transmit precoding in the downlink of TDD-CDMA systems due to the multiple advantages over MUD, including the simple implementation of power control and user scheduling, and the reduction of the power consumption and complexity at the mobile unit. Finally, in conjunction with the precoding techniques we have proposed very low-complexity opportunistic user scheduling algorithms to maximize the utilization of the wireless resources. Simulations results have shown that the proposed algorithms obtain nearly optimal performance.

References


Figure 1: Chip-wise precoding vs. bit-wise precoding: CDF of the required power $P_T$ at the transmitter to achieve $\gamma_k = 10$dB, $\forall k$. Spreading gain $N = 31$, $K = 15$ users.
Figure 2: Chip-wise precoding with matched-filter vs. bit-wise precoding with RAKE receiver: CDF of the required power $P_T$ at the transmitter to achieve $\gamma_k = 13$dB, $\forall k$. Spreading gain $N = 31$, $K = 22$ users.
Figure 3: Linear precoding vs. linear MUD: CDF of the required power $P_T$ at the transmitter to achieve $\gamma_k = 13\text{dB}$, $\forall k$. Spreading gain $N = 31$, $K = 15$ users.
Figure 4: Linear precoding vs. linear MUD: CDF of the required power $P_T$ at the transmitter to achieve $\gamma_k = 10\,\text{dB}$, $\forall k$. Spreading gain $N = 31$, $K = 27$ users.
Figure 5: Linear precoding vs. linear MUD: BER performance with random spreading sequences. Spreading gain $N = 31$, $K = 15$ and $K = 27$ users.
Figure 6: Linear precoding vs. linear MUD: BER performance with Gold spreading sequences. Spreading gain $N = 31$, $K = 15$ and $K = 27$ users.
Figure 7: Average number of users allocated with respect to the total transmit power. Random codes, spreading gain $N = 8$, $U = 12$ available users in the network, and target SINR $\gamma = 12$dB.
Figure 8: CDF of the required total power at the transmitter to allocate the best $K = 4$ users with target SINR per user $\gamma = 12$dB. Hadamard codes, spreading gain $N = 8$, $U = 8$ available users.
Figure 9: CDF of the required total power at the transmitter to allocate the best $K = 4$ users with target SINR per user $\gamma = 12$dB. Random codes, spreading gain $N = 8$, $U = 16$ available users.